A Sensorless Control for PMSM.

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Introduction
PMSM is a kind of electrical machines that due to the absence of external rotor excitation exhibits very high efficient operation.

The state–feedback control problem is relatively simple to solve and several alternatives can be found in the literature.

Concerning sensorless control some partial solutions can be found dealing with speed estimation and unknown load–torque.

To the best of the authors’ knowledge any asymptotically stable sensorless control scheme for PMSM has been reported.
To carry out an integration work in order to state the stability properties of a control system composed by the motor itself, a position observer and a speed/load torque observer.

**Contribution**
By means of Lyapunov’s indirect method, local stability proof of a complete sensorless control scheme for PMSM.
PMSM model
The classical $\alpha\beta$ model of the unsaturated non-salient PMSM is given by

$$L \frac{d i_{\alpha\beta}}{dt} = -R_s i_{\alpha\beta} + \omega \Phi \begin{bmatrix} \sin \theta \\ -\cos \theta \end{bmatrix} + v_{\alpha\beta}$$

with mechanical dynamic

$$\dot{\theta} = \omega$$
$$J \ddot{\omega} = \tau - \tau_L$$
$$\tau = n_p \Phi (i_\beta \cos \theta - i_\alpha \sin \theta)$$

where

$i_{\alpha\beta}, v_{\alpha\beta}$ – measurable
$\theta, \omega$ – unmeasurable
$L, R_s, \Phi, n_p$ – known
$\tau_L$ – unknown but constant
Considering the transformation

\[
(\cdot)_{dq} = e^{-\mathcal{J} \theta}(\cdot)_{\alpha\beta} = \begin{bmatrix} \rho_{\alpha} \mathcal{I} & \rho_{\beta} \mathcal{J}^T \end{bmatrix}(\cdot)_{\alpha\beta}, \quad \mathcal{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},
\]

it is obtained the \(dq\) model

\[
L \frac{di_{dq}}{dt} = -(R_s I_2 + \omega L \mathcal{J})i_{dq} - \begin{bmatrix} 0 \\ \Phi \omega \end{bmatrix} + v_{dq}
\]

\[
J \dot{\omega} = n_P \Phi i_q - \tau_L
\]

\[
\dot{\theta} = \omega
\]

**Remark:** The first three equations are decoupled from the position. However, all the state is not measurable under sensorless operation since \(y = e^{\mathcal{J} \theta} i_{dq}\).
Consider a system of the form

\[ \dot{x} = f(x, t) + g(x)u \]

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) \((m < n)\) and \( g(x) \) is assumed full rank.

Under the IDA–PBC perspective, the purpose is to find \( u(x) \) such that

\[ \dot{x} = F_d(x) \nabla_x H_d(x) \]

where \( F_d(x) + F_d^T(x) \leq 0 \) and \( x^* = \arg\min H_d(x) \).

The rational behind this choice is that

\[ \dot{H}_d = -\nabla_x H_d(x)^T [F_d(x) + F_d^T(x)] \nabla_x H_d(x) \leq 0 \]

The full information control

\[
V^{FI} = \begin{bmatrix}
-\frac{L}{\Phi n_p} \tau_L \omega \\
-\frac{R}{\Phi n_p} \tau_L + \Phi \omega^*
\end{bmatrix}
\]

globally asymptotically stabilizes the equilibrium point

\[
x^* = \begin{bmatrix} 0 & L & x_3^* \end{bmatrix}^T
\]

The required close-loop structure is achieved with

\[
F_d(x) = \begin{bmatrix}
-\frac{R}{\Phi} & \frac{Ln_p}{J} x_3 & 0 \\
-\frac{Ln_p}{J} x_3 & -R & -\Phi \\
0 & \Phi & 0
\end{bmatrix}
\]

Remark: It is possible to add more damping by feeding back the stator currents.

Under the assumption that all the states are measurable, the structure of the control law is given by

\[ V_{\alpha\beta}^{FI} = \begin{bmatrix} \rho_\alpha I & \rho_\beta J \end{bmatrix} V^{FI} \]

Under sensorless operation and assuming that the load torque \( \tau_L \) is unknown, the control law is given by

\[ V_{\alpha\beta} = \begin{bmatrix} \hat{\rho}_\alpha I & \hat{\rho}_\beta J \end{bmatrix} \hat{V} \]

where

\[ \hat{V} = \begin{bmatrix} -\frac{L}{n_p \Phi} \hat{\tau}_L \hat{\omega} \\ \Phi \omega^* + \frac{R}{n_p \Phi} \hat{\tau}_L \end{bmatrix} \]
Position observer
Notice that under the assumption of known parameters

$$\dot{\lambda}_s = -R_s i_{\alpha\beta} + v_{\alpha\beta} = y; \quad \|\lambda_s - i_{\alpha\beta}\|^2 = \Phi^2$$

are measurable signals.

Under these conditions, it is possible to propose a gradient descent–type flux observer of the form

$$\dot{\hat{\lambda}}_s = y + \gamma \left( \hat{\lambda}_s - L i_{\alpha\beta} \right) \left( \Phi^2 - \|\hat{\lambda}_s - i_{\alpha\beta}\|^2 \right)$$

Hence, it is possible to recover the position dependent functions

$$\hat{\rho}_{\alpha\beta} = \frac{1}{\Phi} \left( \hat{\lambda}_s - i_{\alpha\beta} \right)$$

---

Property. The observer guarantees that the ball

\[ \left\{ \tilde{\lambda}_s \mid \| \tilde{\lambda}_s \| \leq 2\Phi \right\} \]

with \( \tilde{\Lambda}_s = \hat{\lambda}_s - \lambda_s \) is globally asymptotically stable.

Property. If \( \tilde{\rho}_{\alpha\beta} = \hat{\rho}_{\alpha\beta} - \rho_{\alpha\beta} \) the observer is equivalent to

i) \[
\dot{\tilde{\rho}}_{\alpha\beta} = -\gamma \Phi^2 (\| \tilde{\rho}_{\alpha\beta} \|^2 + 2\tilde{\rho}_{\alpha\beta}^T \rho_{\alpha\beta}) (\tilde{\rho}_{\alpha\beta} + \rho_{\alpha\beta})
\]

ii) \[
\dot{\hat{\rho}}_{\alpha\beta} = -\gamma \Phi^2 (\| \hat{\rho}_{\alpha\beta} \|^2 - 1) \hat{\rho}_{\alpha\beta} + \omega \mathcal{J} \rho_{\alpha\beta}
\]
Speed and load torque observer
Consider the mechanical dynamics
\[
\dot{\omega} = \frac{n_p \Phi}{J} i_{\alpha\beta}^T J (\hat{\rho}_{\alpha\beta} - \tilde{\rho}_{\alpha\beta}) - \frac{\tau_L}{J}
\]
\[
\dot{\tau}_L = 0
\]
together with
\[
\dot{\hat{\rho}}_{\alpha\beta} = -\gamma \Phi^2 (\|\hat{\rho}_{\alpha\beta}\|^2 - 1) \hat{\rho}_{\alpha\beta} + \omega J (\hat{\rho}_{\alpha\beta} - \tilde{\rho}_{\alpha\beta})
\]

**Remark:** Notice that if \( \tilde{\rho}_{\alpha\beta} = 0 \) then the system is linear with respect the unmeasurable state \( \omega \) and \( \tau_L \) while \( \hat{\rho}_{\alpha\beta} \) can be viewed as a measurable output.

**Remark:** Under these conditions it is possible to apply Immersion and Invariance arguments for the observer design.

\[\text{4Shah D. et. al.: "Speed and Load Torque Observer for Rotating Machines", 48th IEEE Conference on Decision and Control, Shangai, Chine, 2009.}\]
Define

\[
\begin{bmatrix}
\hat{\omega} \\
\hat{\tau}_L
\end{bmatrix} = z + \zeta(\hat{\rho}_{\alpha\beta})
\]

The error dynamic is

\[
\dot{\chi}_{67} = \dot{z} + \dot{\zeta}(\hat{\rho}_{\alpha\beta}) + \begin{bmatrix}
\frac{\tau_L}{J} - \frac{n_p\Phi_i T_{\alpha\beta}}{J} & \frac{n_p\Phi_i T_{\alpha\beta}}{J} \\
0 & 0
\end{bmatrix}
\]

Consider

\[
\zeta(\hat{\rho}_{\alpha\beta}) = \begin{bmatrix}
a_1 \\
-a_2
\end{bmatrix} \arctan\left(\frac{\hat{\rho}_{\beta}}{\hat{\rho}_{\alpha}}\right)
\]

and the observer given by

\[
\dot{z} = \begin{bmatrix}
-a_1 & -\frac{1}{J} \\
a_2 & 0
\end{bmatrix} \hat{x}_{67} + \begin{bmatrix}
\frac{n_p\Phi_i T_{\alpha\beta}}{J} & \frac{n_p\Phi_i T_{\alpha\beta}}{J} \\
0 & 0
\end{bmatrix}
\]
The error dynamic finally takes the form

\[ \dot{\chi}_{67} = \begin{bmatrix} -a_1 & -\frac{1}{J} \\ a_2 & 0 \end{bmatrix} \chi_{67} \]

**Remark:** Notice that \( \zeta(\hat{\rho}_{\alpha\beta}) \) recovers an estimate of the position, which is periodic changing from \(-\pi\) to \(\pi\), or vice versa, each revolution. Then it is not differentiable.

**Remark:** The complete error dynamic is given by

\[ \dot{\chi}_{67} = \begin{bmatrix} -a_1 & -\frac{1}{J} \\ a_2 & 0 \end{bmatrix} \chi_{67} + \begin{bmatrix} a_1 \\ -a_2 \end{bmatrix} \delta(nT) - \frac{\omega}{\|\hat{\rho}_{\alpha\beta}\|^2} \begin{bmatrix} a_1 \\ -a_2 \end{bmatrix} \hat{\rho}_{\alpha\beta}^T \hat{\rho}_{\alpha\beta} + \begin{bmatrix} \frac{n_P \Phi \cdot T}{J} \\ 0 \end{bmatrix} \hat{\rho}_{\alpha\beta} \]

\[ \hat{\rho}_{\alpha\beta} \]
Closed Loop Analysis

Controller error
Position observer error
Speed observer error
Main result

Implementable Diagram

Simulation results

Concluding remarks
Consider the closed–loop error vector

\[
\chi = \begin{bmatrix}
i_{\alpha \beta} - i_{\alpha \beta}^* \\
\omega - \omega^* \\
\hat{\rho}_{\alpha \beta} - \rho_{\alpha \beta} \\
\hat{\omega} - \omega \\
\hat{\tau}_L - \tau_L
\end{bmatrix} = \begin{bmatrix}
\chi_{13} \\
\chi_{45} \\
\chi_{67}
\end{bmatrix}
\]

Substituting the implementable controller and expanding terms

\[
\dot{\chi}_{13} = A_{11}\chi_{13} + A_{12}\chi_{45} + A_{13}\chi_{67} + \Gamma(\chi)
\]

where \(A_{11} = F_d(x^*)Q\) is a Hurwitz matrix with

\[
F_d(x^*) = \begin{bmatrix}
-R & \frac{L_{np}}{J} x_3^* & 0 \\
\frac{L_{np}}{J} x_3^* & -R & -\Phi \\
0 & \Phi & 0
\end{bmatrix};
Q = \begin{bmatrix}
L_s^{-1} & 0 \\
0 & \frac{n_p}{J}
\end{bmatrix}
\]

while \(\Gamma(0) = 0, \nabla_\chi \Gamma(\chi) = 0\) when \(\chi = 0\) and
Concerning the position observer error

\[
\dot{\chi}_{45} = A_{22} \chi_{45} + \Gamma(\chi)
\]

where

\[
A_{22} = \begin{bmatrix}
-2\gamma \Phi^2 & \frac{n_p}{J} x_3^* \\
-\frac{n_p}{J} x_3^* & 0
\end{bmatrix}
\]

is a Hurwitz matrix and \(\Gamma(\chi)\) with the same properties as above.
The speed and load torque observer error is given by:

\[ \dot{\chi}_{67} = A_{23} \chi_{45} + A_{33} \chi_{67} + \Gamma(\chi) + \begin{bmatrix} a_1 \\ -a_2 \end{bmatrix} \delta(nT) \]

where

\[ A_{23} = \begin{bmatrix} -\frac{n_p}{J} \left( a_1 x_3^* - \frac{\Phi x_2^*}{L} \right) & -\frac{n_p}{J} \frac{\Phi x_1^*}{L} \\ \frac{n_p}{J} a_2 x_3^* & 0 \end{bmatrix} \]

\[ A_{33} = \begin{bmatrix} -a_1 & -\frac{1}{J} \\ a_2 & 0 \end{bmatrix} \]

and \( \Gamma(\chi) \) as in the previous cases.

Notice that the matrix \( A_{33} \) is Hurwitz for \( a_1 > 0 \) and \( a_2 > 0 \).
Consider the PMSM model. Assume

**A.1** The only available for measurement variables are stator currents and voltages.

**A.2** The load torque is an unknown *constant* perturbation.

**A.3** All the parameters are known.

Under these conditions, the output–feedback control equipped with the position observer and the speed/load–torque observer guarantees that the closed–loop equilibrium point $\chi = 0$ is locally asymptotically stable.

*Proof.*

\[
\dot{\chi} = A\chi + \Gamma(\chi)
\]

where

\[
A = \begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
0 & A_{22} & 0 \\
0 & A_{32} & A_{33}
\end{bmatrix}
\]
Implementable Diagram
Implementable Diagram

Introduction
PMSM model
Controller Design
Position observer
Speed and load torque observer
Closed Loop Analysis
Implementable Diagram
Implementable Diagram
Simulation results
Concluding remarks
Simulation results

- Constant speed
- Varying speed
- Zero speed
- Varying load–torque

Concluding remarks
Constant speed

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  - Constant speed
  - Varying speed
  - Zero speed
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- Concluding remarks
Constant speed

A graph showing estimated load torque over time for different operations:
- Constant speed
- Varying speed
- Zero speed
- Varying load–torque
Constant speed

Estimated cos/sin vs Time [s]

-1.5 to 1.5

Time [s]: 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5

Estimated cos/sin values for different conditions:
- Constant speed
- Varying speed
- Zero speed
- Varying load–torque

Concluding remarks
Constant speed

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Varying speed

Layout

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Constant speed

Varying speed

Zero speed

Varying load–torque

Concluding remarks
Varying speed

Estimated load torque [Nm]

Time [s]
Zero speed
Varying load–torque
Concluding remarks
It has been presented a sensorless control of PMSM with proved asymptotic stability properties.

A deep analysis concerning the region of attraction of the equilibrium point is required.

Current research is carried out for finding a filter that eliminates the peaks, e.g. notch filters.

A topic of interest is the extension of this results to other types of electrical machines.