Optimal secondary control for DC microgrids

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Context of the study
Parallel interconnection of DC/DC converters

$S_1 \quad L_1 \quad i_1$

$DG_1 \quad Buck \; Converter \; 1$

$\frac{1}{u_1} \quad v$

$DG_m \quad Buck \; Converter \; m$

@implementation

- $v_{in,1}$
- $v_{in,m}$
- $S_1$
- $D_1$
- $L_1$
- $i_1$
- $DG_1$
- $Buck \; Converter \; 1$
- $PWM$
- $DG_m$
- $Buck \; Converter \; m$
- $PWM$
- $C$
- $R$
- $v$
- $u_1$
- $i_1$
- $i_m$
- $v_{in,1}$
- $v_{in,m}$
- $S_m$
- $D_m$
- $L_m$
- $i_m$
- $DC \; BUS$
- $Load$
Motivations:

- simplified model of DC Microgrid
- topology of multiphase buck converters
The related control problem

- “topology dependent”:
  - stability of $m$ parallel converters can depend on $m$ (Thottuvelil and Verghese 1997)

- underspecified:

  $$v = V_{\text{ref}} \iff \sum_k i_k = V_{\text{ref}}/R$$  \hspace{1cm} (1)

  - How to assign $i_k$ such that (1)?
Balanced current-sharing (desired $i_k$ identical):

- most wide-spread strategy
- for heterogeneous converters, not the optimal strategy w.r.t. power losses
Contribution

New control scheme achieving **optimal current-sharing at the steady-state**

- arbitrary and unknown **load**
- valid for an **arbitrary number of converters**
- **experimental validation** are provided
Control problem statement
Hierarchical control scheme

(derived from (Guerrero et al. 2011))

- **Primary control:**
  \[(i_k, \Delta v_k, v) \mapsto d_k\]
  - Ensures stability for all \( R \) by relying only on \( v \) and \( i_k \) (local feedback)
Hierarchical control scheme

- **Secondary control**: 
  \[ V_{\text{ref}} - v \mapsto w_k \]

  - Corrects voltage deviation so that \( v(t) \rightarrow V_{\text{ref}} \)
  - Defines current-sharing policy
Hierarchical control scheme

- Communication between layers
  - unidirectional
  - low bandwidth
  - not required for stability
Problem: Find:

1. a stabilizing primary controller delivering $d_k$
2. a secondary control law computing $w_k$ and imposing
   \begin{itemize}
   \item $v^* = V_{\text{ref}}$
   \item $i^* = i_{\text{opt}}$ which minimizes power losses for $v = V_{\text{ref}}$
   \end{itemize}

for all constant $R$.

(Notation: $i^*$ refers to $\lim_{t \to +\infty} i(t)$)
Experimental benchmark
Experimental setup
Considered electrical model

Assumptions:

1. average (ripple-free) continuous-time model
2. continuous conduction mode
3. $R_{F,k} = R_{S,k}$ for all $k$
∀k, \[ L_k \frac{d i_k}{d t} = -v - (R_{L,k} + R_{F,k})i_k \]
\[ + (V_{in,k} + V_{F,k})d_k - V_{F,k} \]  \hspace{1cm} (2a)

\[ C(R + R_C) \frac{d v}{d t} - CRR_C \frac{d i_T}{d t} = Ri_T - v \]  \hspace{1cm} (2b)

where \[ i_T := \sum_k i_k \]  \hspace{1cm} (2c)
Localized primary control
Proposed primary control

Define

\[ d_k = \frac{1}{V_{F,k} + V_{in,k}} \left( \alpha_k \Delta v_k - R_d i_k + V_{F,k} + v \right) \]

where

\[ R_d := \beta_k - R_{L,k} - R_{F,k} \]

(recall that \( \Delta v_k = (V_{\text{ref}} + w_k) - v \))

If \( \alpha_k > 0 \) and \( \beta_k > 0 \), then stability (of the whole system) is ensured for all \( R \).
Sketch of proof

- Closed-loop current dynamic reads:
  \[ L_k \frac{di_k}{dt} = \alpha_k \Delta v_k - \beta_k i_k. \]  
  (recall that \( \Delta v_k = (V_{\text{ref}} + w_k) - v \))

- The Lyapunov function
  \[ V(i, v) = x^\top \text{diag}\left\{ \frac{L_1}{\alpha_1}, \cdots, \frac{L_m}{\alpha_m}, C \right\} x \]

  with
  \[ x = \begin{bmatrix} i_1 & \cdots & i_m & v - R_C(i_T - v/R) \end{bmatrix}^\top \]

  is strictly decreasing along closed-loop system trajectories.
Selection of $\alpha_k$s and $\beta_k$s

- $\beta_k$: Define time constant $\tau_k = \frac{L_k}{\alpha_k} = \frac{L_k}{\beta_k/\alpha_k}$
- $\alpha_k$: For given $\beta_k$, assign steady-state deviation since $v^* = (V_{ref} + w_k) \frac{R}{R + \beta_k/\alpha_k}$. 
Optimal secondary control
Characterization of the optimal equilibrium

• Power losses in $k$-th branch:

$$p_k = (R_{F,k} + R_{L,k})i_k^2 - V_{F,k}i_k d_k + (V_{F,k} + t_{SW,k} f_s V_{in,k})i_k$$

• Steady-state expression of $p_k$ for $v = V_{ref}$:

$$p_k^* = r_{1,k}(i_k^*)^2 + r_{2,k}i_k^*$$

with

$$r_{1,k} = V_{in,k}(R_{F,k} + R_{L,k})/(V_{in,k} + V_{F,k})$$

$$r_{2,k} = -V_{F,k}(V_{F,k} + V_{ref})/(V_{in,k} + V_{F,k}) + V_{F,k} + f_s t_{SW,k} V_{in,k}$$
• Optimal steady-state current distribution:

\[ i_{\text{opt}} := \arg \min_{i^*} \sum_k p_k^*(i_k^*) \quad \text{s.t.} \quad v^* = V_{\text{ref}} \]

\[ = (1_m - \Gamma \Psi r_1) V_{\text{ref}} / (mR) - \Gamma \Psi r_2 / 2, \]

where

\[ \Psi := \left( \begin{array}{c} \Gamma^T \begin{bmatrix} r_{1,1} & 0 & \cdots & 0 \\ 0 & r_{1,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{1,m} \end{bmatrix} \Gamma \right)^{-1} \Gamma^T \]

\[ \Gamma := \begin{bmatrix} 1 & 0 \\ -1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots \\ 0 & \cdots & -1 \end{bmatrix} \in \mathbb{R}^{m \times (m-1)}; \quad 1_m := \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^m \]
\[ i_1 (A) \]

\[ i_2 (A) \]

\[ i_1 + i_2 = V_{\text{ref}} / R \]

\[ i_1 = i_2 \]

\[ (i_{\text{opt},1}, i_{\text{opt},2}) \]
Optimal secondary control

\[
\begin{align*}
\dot{z} & = \epsilon (V_{\text{ref}} - v) \\
w & = Fz + H
\end{align*}
\]

\( z \in \mathbb{R} \ ; \ F \in \mathbb{R}^m \ ; \ H \in \mathbb{R}^m \)
If $\epsilon > 0$ is sufficiently small and

\[
F = \text{diag} \kappa^{-1} (1_m - \Gamma \psi r_1), \\
H = -\text{diag} \kappa^{-1} \Gamma \psi r_2/2
\]

with

\[
\kappa := \left[ \frac{\alpha_1}{\beta_1} \cdots \frac{\alpha_m}{\beta_m} \right]^\top
\]

then stability is preserved and $v^* = V_{\text{ref}}$ and $i^* = i_{\text{opt}}(R)$, for all $R$.

($F$ and $H$ are independent of $R$)
Sketch of proof

• $\epsilon$ small = frequency separation
  • $\Rightarrow$ primary control at the steady-state
• Dynamic of $z$ can be approximated by

$$\dot{z} \approx \epsilon \chi_R \left( \frac{V_{\text{ref}}}{R} - mz \right)$$

where

$$\chi_R := \frac{1}{\frac{1}{R} + \frac{1}{m\kappa}}$$

• As $\epsilon \chi_R m > 0$, dynamics of $z$ is stable:
  • $\Rightarrow v^* = V_{\text{ref}}$
  • $\Rightarrow i^* = i_{\text{opt}}(R)$
Balanced current-sharing can be imposed via:

\[
F = \text{diag} \kappa^{-1} 1_m \\
H = 0
\]
Experimental results
\[ R = 1 \, \Omega \]
\[ R_1 = 12 \, \Omega \]

\[ i_1 (A) \]
\[ i_2 (A) \]

\[ p_T(i_1, i_2) \]

- \[ i_1 + i_2 = \frac{V_{ref}}{R} \]
- \[ i_1 = i_2 \]
- \[ (i_{opt,1}, i_{opt,2}) \]
Conclusions and perspectives
Hierarchical control scheme with experimental validation

1. new primary control layer
2. new secondary control layer
   • optimal steady-state current-sharing policy
   • cheap and robust communication substructure
     • no current measurements
     • unidirectional communication
     • low-bandwidth
     • not required for stability
   • arbitrary number of converters
• optimal synthesis of $\alpha$ and $\beta$ (distributed control problem):
  Find structured state-feedback of the form

\[
d = \begin{bmatrix}
\star & 0 & \ldots & 0 & \star \\
0 & \star & \ddots & \vdots & \ddots \\
\vdots & \ddots & \ddots & 0 & \vdots \\
0 & \ldots & 0 & \star & \star \\
0 & \ldots & 0 & \star & \star \\
\end{bmatrix}
\begin{bmatrix}
i_1 \\
\vdots \\
i_m \\
v
\end{bmatrix}
\]

• take transmission line impedance into account
References