Effect of Saliency on a Position Observer for Permanent Magnet Synchronous Motors

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Main Contribution Determination of operating regimes for “good behavior” of a position observer for permanent magnet synchronous motors (PMSM) in spite of saliency.
Background

- A globally convergent position observer for non-salient PMSM
- Contract European Embedded Control Institute–Schneider Electric
  - Sabbatical year of Harish Pillai (2012)
  - Post-doc Jose Romero (2013–2014)

Main question: Effect of saliency
Modeling of a salient PMSM

(In $\alpha\beta$ frame) Faraday's and Ohm's law

$$\dot{\lambda} + Ri = v$$

where $\lambda, i, v \in \mathbb{R}^2$ are flux, current and voltage and $R > 0$ is stator resistance.

Flux linkages

$$\lambda = [L_s I_2 + L_g D(\theta)]i + \lambda_m c(\theta)$$

where

$$D(\theta) := \begin{bmatrix} \cos 2n\theta & \sin 2n\theta \\ \sin 2n\theta & -\cos 2n\theta \end{bmatrix}, \quad c(\theta) := \begin{bmatrix} \cos n\theta \\ \sin n\theta \end{bmatrix},$$

$\theta \in \mathbb{S}$ is rotor angle, $n > 0$ the number of pole pairs, $L_s > 0, L_g$ are the stator and “saliency” inductances and $\lambda_m > 0$ the permanent magnet flux.
cont’d

Mechanical coordinates, Newton’s law

\[ J\dot{\omega} = \tau_e - \tau_L, \quad \dot{\theta} = \omega, \]

where \( J \) rotor inertia, \( \tau_L \in \mathbb{R} \) is load torque and the electrical torque is

\[ \tau_e = \frac{\partial}{\partial \theta} \left[ \frac{1}{2} i^\top D(\theta)i + \lambda_m i^\top c(\theta) \right]. \]

\( \alpha\beta \) to \( dq \)-coordinates:

\[ \begin{pmatrix} f_d \\ f_q \end{pmatrix} = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix} \begin{pmatrix} f_\alpha \\ f_\beta \end{pmatrix} \]

\( dq \) model with \( L_s = \frac{1}{2}(L_d + L_q), \quad L_g = \frac{1}{2}(L_d - L_q) \)

\[ \begin{align*}
L_d \frac{di_d}{dt} &= -Ri_d + \omega L_q i_q + v_d \\
L_q \frac{di_q}{dt} &= -Ri_q - \omega L_d i_d - \omega \lambda_m + v_q \\
J \frac{d\omega}{dt} &= n\lambda_m i_q + n(L_d - L_q) i_d i_q - \tau_L.
\end{align*} \]
If $L_g = 0$, $\lambda = L_s i + \lambda_m c(\theta)$. Hence, $\theta = \tan^{-1} \left( \frac{\lambda_\beta - L_s i_\beta}{\lambda_\alpha - L_s i_\alpha} \right)$.

Now, from $\dot{\lambda} = -R_i + v$ it is clear that $\dot{\lambda} = -R_i + v$ estimates $\lambda$ up to a constant due to the unknown initial conditions.

Idea: Add to $\dot{\lambda}$ a correction term in the direction of the negative of the gradient of

$$J(\dot{\lambda}) := (\lambda_m^2 - |\dot{\lambda} - L_s i|^2)^2,$$

which is computable, because

$$\frac{\partial}{\partial \lambda} J(\dot{\lambda}) = -4(\dot{\lambda} - L_s i)(\lambda_m^2 - |\dot{\lambda} - L_s i|^2).$$

This leads to

$$\dot{\lambda} = v - R_i + \gamma(\dot{\lambda} - L_s i)(\lambda_m^2 - |\dot{\lambda} - L_s i|^2),$$

where $\gamma > 0$ is a scaling factor.
P1 (Global stability) For arbitrary speeds, the disk

\{ \tilde{\lambda} \in \mathbb{R}^2 | |\tilde{\lambda}| \leq 2\lambda_\text{m} \},

is globally attractive, where \( \tilde{\lambda} := \hat{\lambda} - \lambda \).

P2 (Exponential stability under persistent excitation) The zero equilibrium is exponentially stable if there exists constants \( T, \Delta > 0 \) such that

\[
\frac{1}{T} \int_{t}^{t+T} \omega^2(s)ds \geq \Delta,
\]

for all \( t \geq 0 \).

P3 (Constant non–zero speed) If the speed is constant and satisfies

\[
|\omega| > \frac{1}{4} \gamma \lambda_m^2,
\]

then the origin is the unique equilibrium and it is globally asymptotically stable.
Ad–hoc inclusion of saliency in the observer design

When $L_g \neq 0$

$$\lambda = [L_s I_2 + L_g D(\theta)]i + \lambda_m c(\theta)$$

Defining the estimate of $c(\theta) = \text{col}(\cos(n\theta), \sin(n\theta))$ as

$$\hat{c} := \frac{1}{\lambda_m} (\hat{\lambda} - L_s i) \quad (C')$$

the observer may be written as

$$\dot{\lambda} = v - Ri + \gamma(\hat{\lambda} - L_s i)\lambda_m^2 (1 - |\hat{c}|^2).$$

Idea: Incorporate additional terms in the observer.

First option

$$\dot{\lambda} = v - Ri + \gamma(\hat{\lambda} - L_s i - L_g D(\hat{\theta}))\lambda_m^2 (1 - |\hat{c}|^2).$$

Second option, replace $(C')$ by

$$\hat{c} := \frac{1}{\lambda_m} (\hat{\lambda} - L_s i - L_g D(\hat{\theta})).$$

Combinations of these two.
Certainty–equivalent velocity and load torque observer

Assuming position known we can compute \( \tau_e \) and design an observer for \( \omega \) and \( \tau_L \) for

\[
\begin{align*}
\dot{\theta} &= \omega \\
J\dot{\omega} &= \tau_e - f\omega - \tau_L,
\end{align*}
\]

where \( f \geq 0 \) is a friction coefficient, with measurable outputs \( y = \text{col}(\sin \theta, \cos \theta) \).

Alternative model of the mechanical equations. Define \( \eta = \text{col}(\omega, \tau/L) \). The system dynamics is

\[
\dot{\eta} = A\eta + \begin{bmatrix} \frac{1}{J} \tau \\ 0 \end{bmatrix}, \quad \dot{y} = \Phi(y)\eta,
\]

where

\[
A := \begin{bmatrix} -f/J & -1 \\ 0 & 0 \end{bmatrix}, \quad \Phi(y) := \begin{bmatrix} y_2 & 0 \\ -y_1 & 0 \end{bmatrix}.
\]

Observation problem non–trivial because \( y \) is a nonlinear function of \( \theta \in \mathbb{S} \).
A globally exponentially stable observer

The fifth–dimensional system

\[
\begin{align*}
\dot{\hat{y}} &= \begin{bmatrix} y_2 \\ -y_1 \end{bmatrix} \hat{\eta}_1 - k_3 r^2 (\hat{y} - y) \\
\dot{\hat{\xi}} &= \begin{bmatrix} \frac{1}{J} \tau e - \frac{f}{J} \hat{\eta}_1 - \hat{\eta}_2 \\ 0 \end{bmatrix} + k_4 [\hat{\eta}_1 (1 - y^\top \hat{y}) + k_3 r^2 (y_1 \hat{y}_2 - \hat{y}_1 y_2)] \\
\dot{r} &= -\frac{k_1}{4} (r - 1) + \frac{k_2}{2k_1} r (1 - y^\top \hat{y})^2, \quad r(0) \geq 1 \\
\hat{\eta} &= \xi + (y_1 \hat{y}_2 - \hat{y}_1 y_2) k_4,
\end{align*}
\]

with \( k_i > 0 \) suitable tuning gains, ensures that \( \hat{y}, \xi, r \) are bounded and

\[
\lim_{t \to \infty} |\hat{\eta}(t) - \begin{bmatrix} \omega(t) \\ \frac{\tau_f}{J} \end{bmatrix}| = 0 \quad (exp).
\]
Simulation: Sensitivity to saliency and parameter uncertainty

Open loop operation, sinusoidal voltages, $q$ weighting factor on $\lambda_m$, $\tau_L = 4.67$

<table>
<thead>
<tr>
<th>$q$</th>
<th>load torque</th>
<th>$(\dot{\omega} - \omega)$</th>
<th>$r$ value after 8 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.04</td>
<td>4.08</td>
<td>-3.5</td>
<td>5</td>
</tr>
<tr>
<td>1.03</td>
<td>4.15</td>
<td>-3.3</td>
<td>2.3</td>
</tr>
<tr>
<td>1.02</td>
<td>4.20</td>
<td>-2.5</td>
<td>1.5</td>
</tr>
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<td>1.01</td>
<td>4.26</td>
<td>-2.0</td>
<td>1.37</td>
</tr>
<tr>
<td>1.00</td>
<td>4.32</td>
<td>-1.2</td>
<td>1.28</td>
</tr>
<tr>
<td>0.99</td>
<td>4.38</td>
<td>-0.5</td>
<td>1.25</td>
</tr>
<tr>
<td>0.98</td>
<td>4.44</td>
<td>+0.3</td>
<td>1.24</td>
</tr>
<tr>
<td>0.97</td>
<td>4.50</td>
<td>+1.0</td>
<td>1.25</td>
</tr>
<tr>
<td>0.96</td>
<td>4.57</td>
<td>+1.7</td>
<td>1.3</td>
</tr>
<tr>
<td>0.95</td>
<td>4.63</td>
<td>+2.5</td>
<td>1.37</td>
</tr>
<tr>
<td>0.94</td>
<td>4.69</td>
<td>+3.5</td>
<td>1.6</td>
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<tr>
<td>0.93</td>
<td>4.75</td>
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<tr>
<td>0.92</td>
<td>4.82</td>
<td>+5.8</td>
<td>5.6</td>
</tr>
</tbody>
</table>
Variation in resistance $R$

Open loop operation, sinusoidal voltages, $q$ weighting factor on $R$, $\tau_L = 4.67$

<table>
<thead>
<tr>
<th>$q$</th>
<th>load torque</th>
<th>$(\dot{\omega} - \omega)$</th>
<th>$r$ value after 8 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.15</td>
<td>5.00</td>
<td>-3.2</td>
<td>3.0</td>
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<tr>
<td>1.10</td>
<td>4.79</td>
<td>-2.5</td>
<td>1.68</td>
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<tr>
<td>1.09</td>
<td>4.75</td>
<td>-2.5</td>
<td>1.60</td>
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<tr>
<td>1.05</td>
<td>4.56</td>
<td>-1.8</td>
<td>1.40</td>
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<tr>
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<td>4.32</td>
<td>-1.2</td>
<td>1.28</td>
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<td>0.95</td>
<td>4.06</td>
<td>-0.6</td>
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<td>0.60</td>
<td>1.66</td>
<td>+2.7</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Simulation comparison of modified position observer

Open loop operation, sinusoidal voltages, $\tau_L = 4.67$.

Steady–state comparison.

Significant improvement of model 2, marginal for others.

Sustained (small) periodic oscillation in $\dot{\theta}$.

Large impact on $\hat{\tau}_L$ and $\hat{\omega}$.

<table>
<thead>
<tr>
<th>model</th>
<th>minimum error of $\eta \theta$ (rad)</th>
<th>maximum error in $\theta$ (rad)</th>
<th>% error in torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.015</td>
<td>0.08</td>
<td>13.63</td>
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<tr>
<td>2</td>
<td>0.01</td>
<td>0.075</td>
<td>13.0</td>
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<tr>
<td>3</td>
<td>0.01</td>
<td>0.09</td>
<td>7.34</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.08</td>
<td>5.66</td>
</tr>
<tr>
<td>5</td>
<td>0.01</td>
<td>0.09</td>
<td>7.34</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td>0.08</td>
<td>5.66</td>
</tr>
</tbody>
</table>
Effect of saliency on position observer

**Proposition** Define the (scaled and rotated) error signals

\[ \xi = -\frac{1}{\lambda_m} \exp(-n\theta J)\tilde{\lambda}. \]

Introduce the time–scale change \( \frac{dt}{d\tau} = \frac{1}{\gamma \lambda^2_m} \). Then,

\[
\frac{d\xi_1}{d\tau} = \Omega \xi_2 - \sigma(\xi)(\xi_1 - i_d^0 - 1) \\
\frac{d\xi_2}{d\tau} = -\Omega \xi_1 - \sigma(\xi)(\xi_2 + i_q^0)
\]

where

\[
\sigma(\xi) := (\xi_1 - i_d^0 - 1)^2 + (\xi_2 + i_q^0)^2 - 1,
\]

with the scaled currents \( i_{dq}^0 := \frac{L_g}{\lambda_m} i_{dq} \) and the scaled speed \( \Omega = \frac{n}{\gamma \lambda^2_m} \omega \).

**Key Remark** The performance degradation is determined by \( L_g \) and \( i_{dq} \)—in steady–state an equilibrium shift.
Stability properties

C1 (Global stability) For arbitrary speeds, the disk
\[ \{ \tilde{\lambda} \in \mathbb{R}^2 | |\tilde{\lambda}| \leq 1 + \sqrt{(i_d^0 + 1)^2 + (i_q^0)^2} \} , \]
is globally attractive.

C2 (Equilibrium at zero) \((0, 0)\) is an equilibrium if and only if
\[ (i_d^0 + 1)^2 + (i_q^0)^2 = 1. \]
Moreover,
- If \(0 < \Omega \leq \frac{1}{2}\), then there are three equilibria and the origin is a stable node.
- If \(\Omega > \frac{1}{2}\), then the origin is the only equilibrium. It is a stable node for \(\Omega \leq 1\) and a stable focus for \(\Omega > 1\).

C3 (Limit cycles) If \((i_d^0 + 1)^2 + (i_q^0)^2 \leq 1/2\), there is an (almost globally) stable limit cycle.

C4 (Equilibria away from zero) If \((i_d^0 + 1)^2 + (i_q^0)^2 > 1\), there might be one or three equilibrium points.
Case \((i_d^0 + 1)^2 + (i_q^0)^2 = 1\) high speed
Case \((i_d^0 + 1)^2 + (i_q^0)^2 = 1\) low speed

\[
x' = K y - (x - a - 1) ((x - a - 1)^2 + (y + b)^2 - 1) \\
y' = -K x - (y + b) ((x - a - 1)^2 + (y + b)^2 - 1)
\]

\[
K = 0.2 \quad a = -0.4 \quad b = 0.8
\]
Case \((i_d^0 + 1)^2 + (i_q^0)^2 > 1\) high speed
Case \((i_d^0 + 1)^2 + (i_q^0)^2 > 1\) low speed

\[
x' = K y - (x - a - 1) ((x - a - 1)^2 + (y + b)^2 - 1)
\]
\[
y' = -K x - (y + b) ((x - a - 1)^2 + (y + b)^2 - 1)
\]

\[K = 0.2 \quad a = 0.5 \quad b = 0.2\]
Case $(i_d^0 + 1)^2 + (i_q^0)^2 < 1/2$: Sustained oscillations

$x' = Ky - (x - a - 1) ((x - a - 1)^2 + (y + b)^2 - 1)$
$y' = -Kx - (y + b) ((x - a - 1)^2 + (y + b)^2 - 1)$

A plot is shown with the following parameters:
- $K = 1$
- $a = -0.5$
- $b = 0.4$

The backward orbit from (-0.22, 1.2) left the computation window.
Ready.
The forward orbit from (1.8, 0.18) -> a nearly closed orbit.
The backward orbit from (1.8, 0.18) left the computation window.
Ready.
Future research

Final objective to propose a sensorless controller for salient PMSM:
- robust to saliency and parameter uncertainty,
- with flux saturation effects,
- comparison of the proposed scheme (or combination) with high–frequency signal injection at low speeds.

The clear understanding of the role of saliency on the position observer can guide us to the redesign of both, the velocity and load torque observers and the energy–shaping controller.

Recent results of robustifying energy–shaping controllers developed with the PhD student Jose Romero: achieved with simple (nonlinear) PI controllers.

Same techniques applicable to observers?

Saturation effects opens a completely new research avenue: use of the new Euler–Lagrange models.

The final test of the performance should be carried out in a specialized experimental rig.