High-Voltage Direct-Current Transmission Systems
— From theory to practice and back

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Outline

1. Introduction: system topology & control architecture
2. Energy–based modeling
3. Power flow analysis
4. PI controllers:
   - Design: a GAS PI passivity–based controller
   - Analysis: stability and performance limitations
5. Overcoming performance limitations
6. Conclusions & future research
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Multiterminal HVDC Transmission Systems

Multiterminal HVDC transmission system

**Definition.** Direct Current (DC) electrical system operating at high voltage (HV), with no loads. It is constituted by *n* terminals that are connected via *ℓ* DC transmission lines.

- AC sources connected to the terminals are associated to MVAC grids or Renewable Energy Sources (RES)
- and interfaced through power converters, that:
  - perform the AC/DC conversion
  - regulate the power flow and/or the voltages
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Background and motivation

Popularity of HVDC transmission is due to the following features

- interconnection of non–synchronous AC grids (B2B connection)
- transmission over long distances (reduced losses)
  - island systems connection
  - integration of *remotely located* RES

but introduces new control problems...

- intermittent nature of RES
- power converters are *highly nonlinear* elements
- no time–scale separation between transmission and generation
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Control architecture: nominal conditions

...and still employs the control structure of traditional AC systems

- Refs calculator: takes as inputs the desired behavior and the power flow and derive the setpoints
- Feasible setpoints (nominal conditions) are stabilized by the inner control, under perfect knowledge of the power flow
Control architecture: perturbed conditions

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Power flow uncertainty or contingencies (perturbed conditions) ⇒ not feasible setpoints. Primary control adapts refs to

- be feasible and so stabilizable by the inner control
- preserve suitable properties (power sharing, voltage stability)
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Energy–based modeling

Power system as interconnection of single components

- Power converters \( (C) \), that are nonlinear elements
- DC transmission lines \( (L) \), as linear \( \pi \)-models
- Interconnection laws described by standard KVL, KCL

Assumptions

A1. Balanced operation of the line voltages
A2. Ideal four–quadrant operation of the converter
A3. Switches can be operated sufficiently fast
    \( \Rightarrow \) Average model of the converter
A4. Synchronized operation of the converters
    \( \Rightarrow \) AC source \( \equiv \) constant amplitude/frequency voltage source
    \( \Rightarrow \) \( dq \)-frame representation \( \equiv \) constant steady–states
Energy–based modeling

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Energy-based modeling

Components modeled as port–Hamiltonian (pH) systems

\[
\dot{x} = (\mathcal{J}(x) - \mathcal{R}(x))\nabla_x \mathcal{H}(x) + G(x)u + E
\]

\[
y = G^\top(x) \nabla_x \mathcal{H}(x),
\]

with: \(x\) the state in physical variables (fluxes, charges), \(\mathcal{J} = -\mathcal{J}^\top(x)\), \(\mathcal{R}(x) \geq 0\), \(\mathcal{H}(x)\) the interconnection, damping and energy functions.

Interconnection laws captured by a graph \(\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})\)

For a graph \(\mathcal{G}\) with set of vertices \(\mathcal{V}\), edges \(\mathcal{E}\) and incidence matrix \(\mathcal{M}\), KCL and KVL are given respectively by

\[
\mathcal{M} I_\mathcal{E} = 0, \quad \mathcal{M}^\top V_\mathcal{V} = V_\mathcal{E},
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with \(I_\mathcal{E}, V_\mathcal{E}\) current and voltages of the edges and \(V_\mathcal{V}\) voltages at the vertices (nodes).
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Energy-based modeling

\[ n = 3 \text{ CONVERTERS: } x_C \in \mathbb{R}^{3n}, u \in \mathbb{R}^{2n} \]

\[ \dot{x}_C = (J_C - R_C) \nabla H_C + g_C(x_C)u + E_C - G_C i_C \]

\[ v_C = G_C^\top \nabla H_C, \]

with \( H_C := \frac{1}{2} x_C^\top Q_C x_C \).

\[ \ell = 2 \text{ LINES: } x_L \in \mathbb{R}^\ell \]

\[ \dot{x}_L = -R_L \nabla H_L + v_L \]

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INTERCONNECTION LAWS

\[
M = \begin{bmatrix}
I_n^T & M \\
-1_n^T & 0_{\ell}^T
\end{bmatrix}
\]

(KCL) \( i_C + M i_L = 0_n, \quad -1_n^T i_C = 0 \)

(KVL) \( v = v_C, \quad M^\top v = v_L \)
A multiterminal HVDC transmission system can be modeled as

\[
\dot{x} = \begin{bmatrix} \dot{x}_C \\ \dot{x}_L \end{bmatrix} = \left[ \begin{array}{ccc} (J_R - R_C) & -G_C M \\ M^T G_C & -R_L \end{array} \right] (J - R) \nabla H + \begin{bmatrix} g_C(x_C) \\ 0 \end{bmatrix} u + \begin{bmatrix} E_C \\ E \end{bmatrix}
\]

(1)

- quadratic energy function \( H := \frac{1}{2} x^T Q x \), \( Q = \text{blockdiag}\{Q_C, Q_L\} \)
- structured (linear) input matrix \( g(x) := [J_d Q x \quad J_q Q x] \)
- topology matrix \( M \) issued by the graph \( G \)

why \( dq \)-frame?

- Constant steady-states \( \Rightarrow \) pure regulation problem
- Control active/reactive power \( \Rightarrow \) control of \( dq \) currents
Energy-based modeling

Port–Hamiltonian modeling

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Passivity as a tool for power flow analysis

Admissible equilibria

**Proposition 1.** The admissible equilibria of (1) are the solution of

\[ 0 = \mathcal{H} \mathcal{C},i(x^*_C,i, x^*_L) \]

\[ x^*_L = C x^*_C. \]

with \( C := (\mathcal{R}_L \mathcal{Q}_L)^{-1} M^\top G_C^\top Q_C, \quad i \in [1, n]. \)

why is this interesting?

- Lines equilibria linearly depend on converters equilibria
- Eqs.(2) are the power flow steady–states equations (PFSSE)
- Defines all steady states achievable by ANY stabilizing controller
Admissible equilibria

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Passivity as a tool for power flow analysis

PFSSE can be used to formulate the problem of refs calculation
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Passivity as a tool for inner-loop control design

Control design:
- define the incremental forms
- find a passive output for the incremental model
- design the controller over the passive output

Incremental formulation
Let $x^*$ an admissible equilibrium point and $u^*$ the equilibrium control, i.e.

$$(x^*, u^*) : (J - R)Qx^* + g(x^*)u^* + E = 0.$$  

We define then

$$\tilde{x} := x - x^*, \quad \tilde{u} := u - u^*, \quad \tilde{H}(x) := \frac{1}{2}\tilde{x}^\top Q\tilde{x},$$  

i.e. the errors and the correspondent (incremental) energy function.
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A passive output for the incremental model

**Proposition 2.** Consider the incremental model of (1). The mapping $\tilde{u} \rightarrow y$ is passive with storage function $\tilde{H}(x)$, with

$$y := g^T(x^*)Qx.$$

why is important to find a passive output?

- the feedback interconnection of two passive systems is passive
- the system can be controlled with PIs
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- the system can be controlled with PIs
Proposition 3. Let an assignable reference $x^*$, verifying the PFSSE. Then the PI controller

$$\dot{z} = -y, \quad u = -K_py + K_iz,$$

globally asymptotically regulates the system (1) to $(x^*_C, Cx^*_C)$ for any positive gains $K_I, K_P$.

- **PI control**: simple, easily implementable, intrinsically robust
- Blockdiag matrices for the gains guarantee decentralization
Passivity as a tool for inner–loop control design

### Proposition 3
Let an assignable reference $x^\star$, verifying the PFSSE. Then the PI controller

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globally asymptotically regulates the system (1) to $(x^\star_C, Cx^\star_C)$ for any positive gains $K_I, K_P$.

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Other controllers reported in literature

Power converter states: $dq$–currents $(i_d, i_q)$, DC voltage $v_C$

1. **PI passivity–based controller** based on the output

   $$ y_d := v_C^* i_d - i_d^* v_C \quad y_q := v_C^* i_q - i_q^* v_C $$

2. **PI dq-currents controller** based on the output

   $$ y_d^I := i_d - i_d^* \quad y_q^I := i_q - i_q^* $$

3. **PI voltage controller** based on the output

   $$ y_d^V := v_C - v_C^* \quad y_q^V := i_q - i_q^* $$
Simulations: a three–terminal benchmark

CONTROL OBJECTIVES

- All terminals are required to keep reactive power near to zero
- Terminals \textit{WF1}, \textit{WF2} regulate the injected (absorbed) active power
- The remaining terminal regulates the DC voltage (slack bus, \textit{SB})
Simulations: a three–terminal benchmark

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**SIMULATED SCENARIO:** From 0 to 5T s, with changes every T s

<table>
<thead>
<tr>
<th></th>
<th>SB</th>
<th>$WF_1$</th>
<th>$WF_2$</th>
<th>SB</th>
<th>$WF_1$</th>
<th>$WF_2$</th>
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<tr>
<td>0</td>
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<td>900</td>
<td>1000</td>
<td>100</td>
<td>142.595</td>
<td>158.951</td>
</tr>
<tr>
<td>T</td>
<td>-1588</td>
<td>900</td>
<td>1800</td>
<td>100</td>
<td>153.650</td>
<td>179.691</td>
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<td>2T</td>
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<td>500</td>
<td>-200</td>
<td>100</td>
<td>109.004</td>
<td>104.004</td>
</tr>
<tr>
<td>3T</td>
<td>905</td>
<td>-400</td>
<td>-200</td>
<td>100</td>
<td>69.419</td>
<td>60.877</td>
</tr>
<tr>
<td>4T</td>
<td>-849</td>
<td>1300</td>
<td>-200</td>
<td>100</td>
<td>128.708</td>
<td>124.532</td>
</tr>
</tbody>
</table>

highly stressed scenario — many power reversals
Simulations: standard PI controllers

Setpoint changes every $T = 4$ s.

Good performances but instability arises under full power flow reversal
Proposition 4. The zero dynamics of the single converter with respect to the output $y^I$ and output $y^V$ are given by the scalar system

$$\dot{\zeta} = -\beta \zeta + \frac{\alpha}{\zeta} - \gamma, \quad \beta, \gamma > 0.$$ 

- the system admits two equilibria, but only one is physically meaningful
- If $\alpha > 0$, i.e. the converter is injecting power the equilibrium is stable
- If $\alpha < 0$, i.e. the converter is absorbing power the equilibrium is unstable

$\Rightarrow$ zero dynamics become unstable when the power flow is reversed
Simulations: passivity–based controllers

Setpoint changes every $T = 200$ s.

Stable for any scenario (as expected) but unacceptably slow transients
Performance limitations analysis: passivity–based controllers

Zero dynamics analysis – Passive output $y$

**Proposition 5.** The zero dynamics of the single converter wrt to the output $y$ is *linear* and *exponentially stable* and converge at a rate

$$\lambda := \frac{1}{2} \frac{P^* - P_{dc}^*}{\mathcal{H}(x_C^*)}.$$  

- $\lambda \approx 0.04$ for HV power converters
- analogous to have a slow zero in linear systems
- increasing the gains, one pole of the closed-loop system is attracted by the slow zero
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$$
\dot{z} = -y, \quad u = -K_P y + K_I z - K_L Q(x - x^*),
$$

globally asymptotically regulates the system (1) to $(x^*_C, Cx^*_C)$ for any positive gains $K_I, K_P, K_L$ verifying

$$
\mathcal{R} + g(x^*)K_P g^T(x^*) + \frac{1}{2} \left[ g(x^*)K_L + K_L^T g^T(x^*) \right] > 0.
$$

- the additional feedback affects only the P-part of the controller
- d.o.f. added to render the Lyapunov derivative negative definite
- the condition can be intended as a constraint for tuning of the gains
Adding a linear feedback to the passivity–based inner loop

Decentralized PI Regulation plus linear feedback

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Adding a linear feedback to the passivity–based inner loop

Inspired by the ubiquitous voltage droop control, we choose $K_L$ such that

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\begin{bmatrix}
u_d \\
u_q
\end{bmatrix} = \begin{bmatrix}-k_{Pd}v_d + k_{Id}z_d - k_D(v_C - v_C^*) \\
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i.e. we introduced an additional linear feedback in the voltage error

$\Rightarrow \exists$ gains $k_P, k_I, k_L$ that ensure

- GAS of the equilibrium point
- decentralization of the controller
- to speed up convergence to $ms$
- drawback: in contrast with conventional droop, stability is lost in perturbed conditions
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Simulations: passivity–based control plus linear feedback

Setpoint changes every $T = 2 \text{ s}$. 

Stable for any scenario with good performances
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Conclusions

- unified framework for modeling using a pH representation
- power flow analysis (assignable behavior)
- PI passivity–based control (PI–PBC) design
- performance and stability analysis of standard PI controllers
- modified PI–PBC to improve performances

[Zonetti, D., Ortega, R., Benchaib, A. Modeling and control of HVDC transmission systems – From theory to practice and back. Control Engineering Practice (2015)]
Future research

▶ extend to lines modeled by PDEs
▶ include AC dynamics to explore AC/DC interactions
▶ design a new GAS primary controller
  ▶ behind the PI–PBC: adaptive control?
  ▶ as a whole with the inner loop
    ⇒ stabilization to an unknown operating point
▶ extend the theory to DC distribution networks interfaced to AC grids ⇒ including loads

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Merci.