On Time Petri Nets and Scheduling

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IRCCyN / École Centrale de Nantes

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Plan I

Plan I

Introduction

Time Petri Nets with Stopwatches (and more)

Abstractions for Scheduling-TPNs

Verifying properties

Conclusion and Future Work

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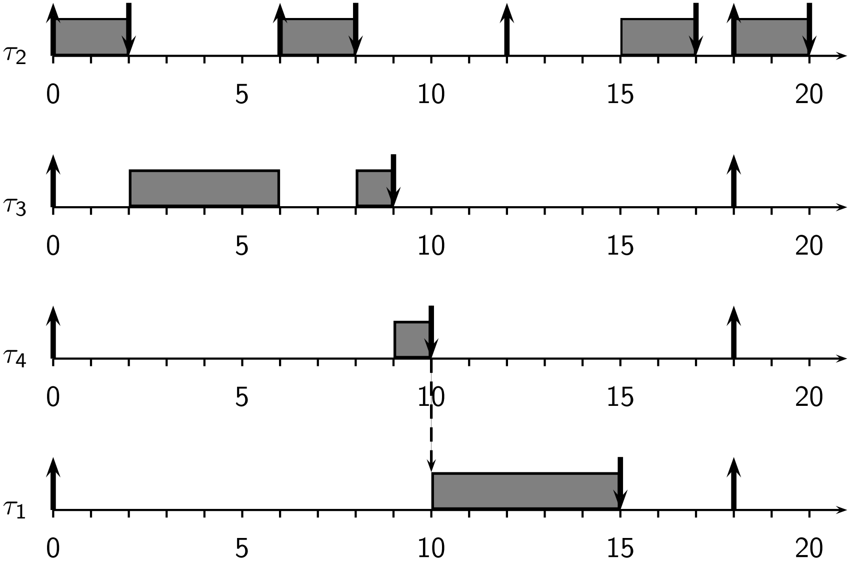
- They are very good with simple settings;
- They are not so good with precedences, timing uncertainties, distribution, . . .

A well-known paradox



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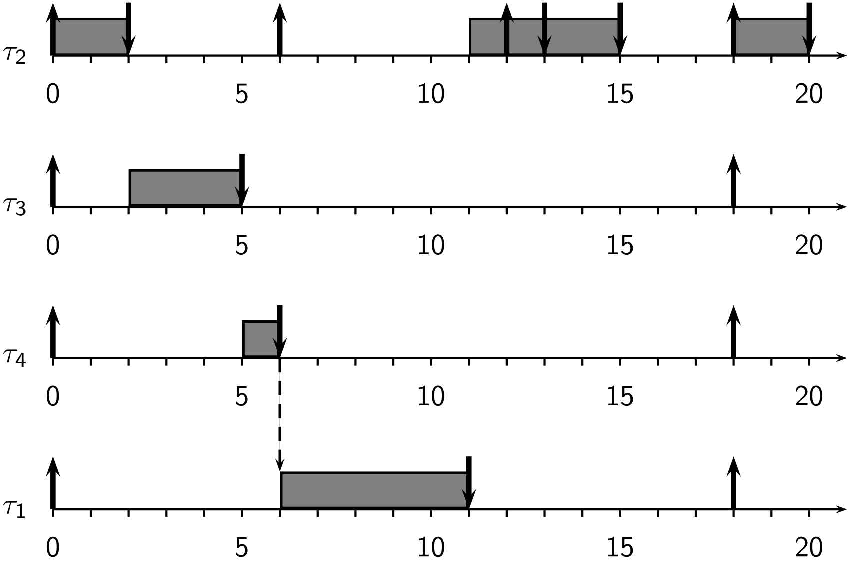
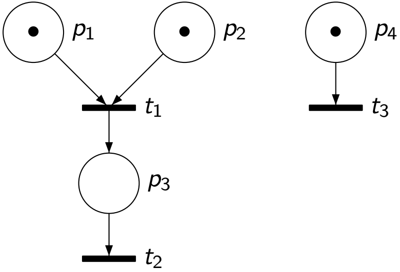
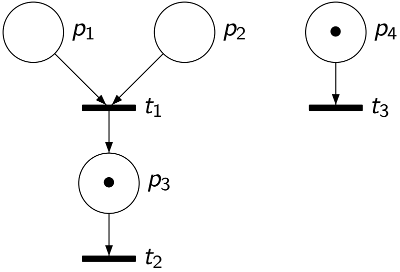


Plan I

Petri Nets

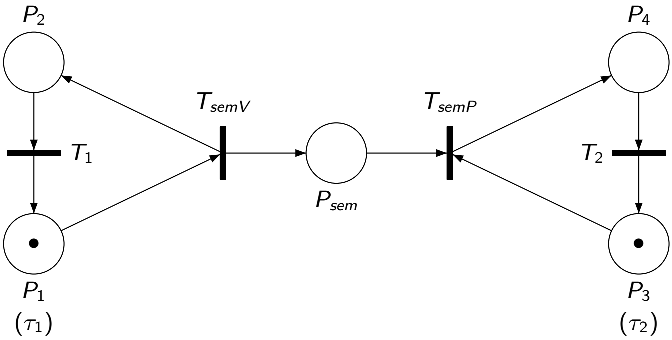


Petri Nets

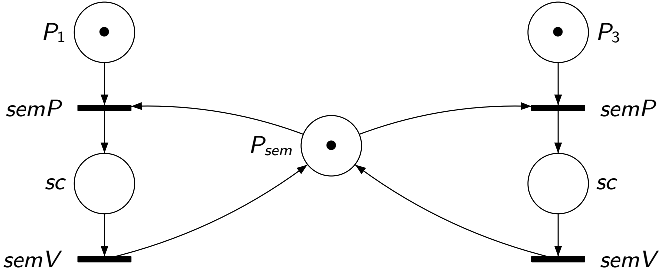


$$\boxed{\text{if } M \geq {}^\bullet t \text{ then } M' = M - {}^\bullet t + t^\bullet}$$

Petri Net Example: Basic Semaphore Synchronization



Petri Net Example: Mutual Exclusion



Petri Net Example: Peterson's Algorithm

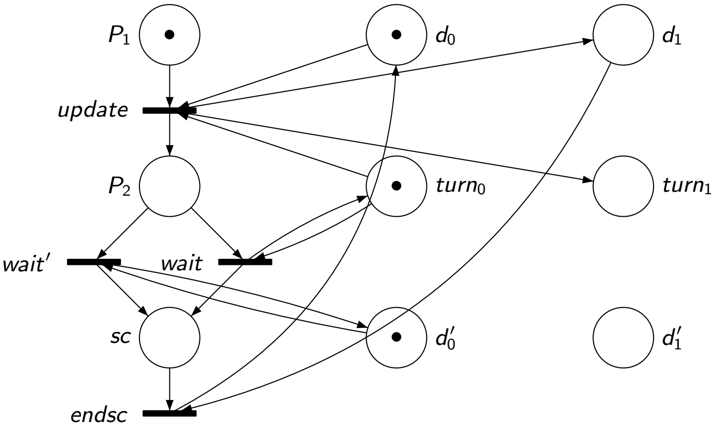
P:

```
d ← false
loop
    <non critical section>
    d ← true
    turn ← 1
    wait(¬d′∨ turn= 0)
    <critical section>
    d ← false
end_loop
```

P':

```
d′ ← false
loop
    <non critical section>
    d′ ← true
    turn ← 0
    wait(¬d∨ turn= 1)
    <critical section>
    d′ ← false
end_loop
```

Petri Net Example: Peterson's Algorithm



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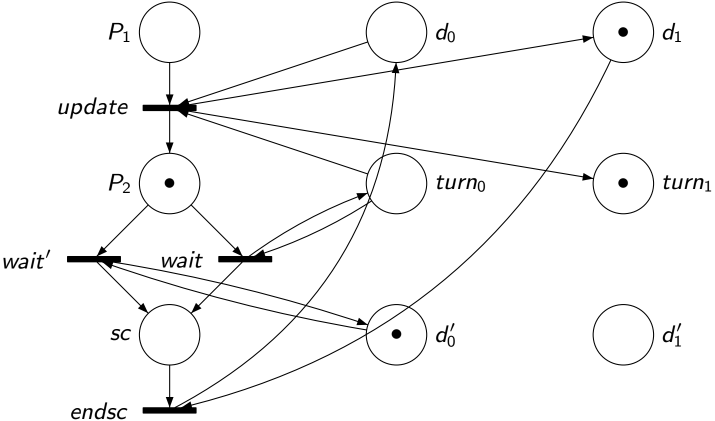

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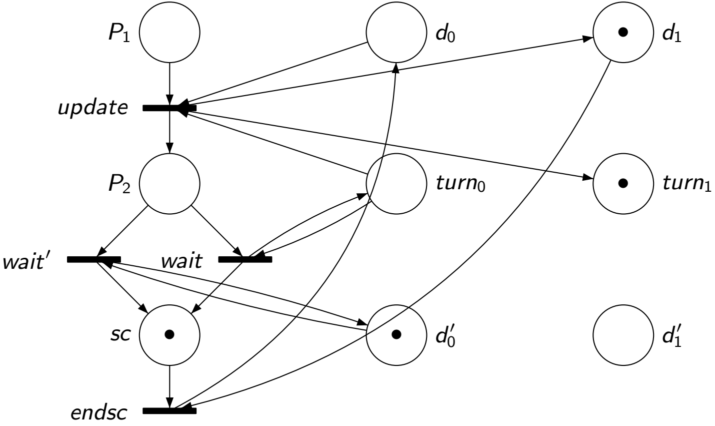

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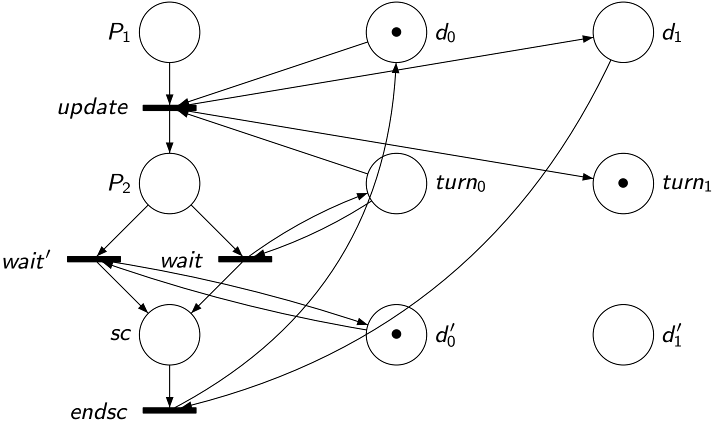
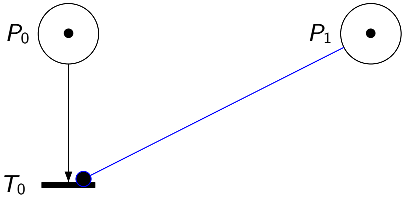


Read Arcs

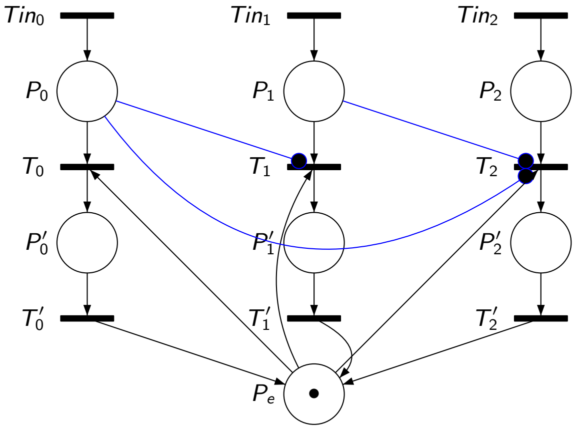
$$\boxed{\text{if } M \geq {}^\bullet t \text{ and } M \geq {}^\diamond t \text{ then } M' = M - {}^\bullet t + t^\bullet}$$

Inhibitor Arcs

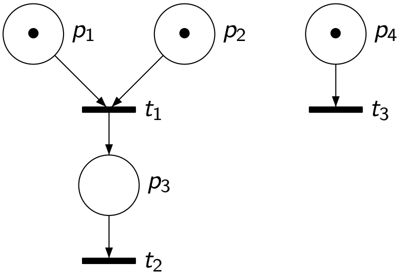


$$\boxed{\text{if } M \geq {}^{\bullet}t \text{ and } M < {}^{\circ}t \text{ then } M' = M - {}^{\bullet}t + t^{\bullet}}$$

Petri Net Example: Fixed Priority Scheduling (non-preemptive)

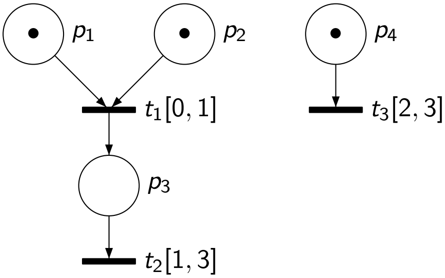


Time Petri Nets



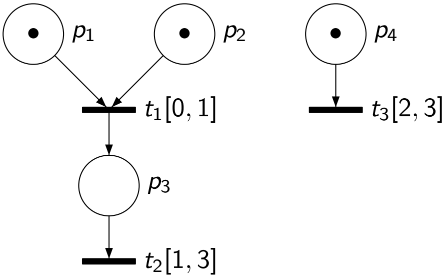
A Petri Net

Time Petri Nets



A **Time** Petri Net

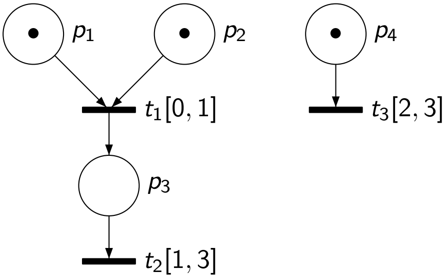
Time Petri Nets



A **Time** Petri Net

$$\left(M_0, \begin{array}{l} t_1 = 0, \\ t_3 = 0 \end{array} \right)$$

Time Petri Nets



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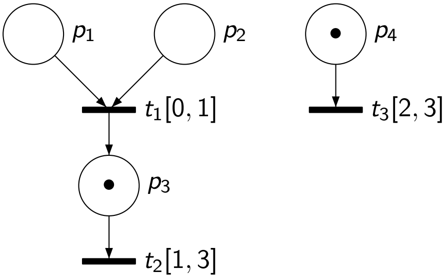
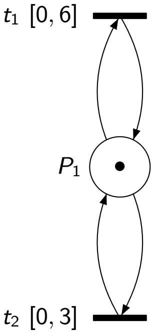
Time Petri Nets



A **Time** Petri Net

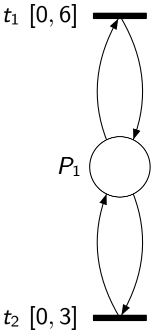
$$\left(M_0,\ \begin{array}{l} t_1 = 0, \\ t_3 = 0 \end{array}\right) \xrightarrow{0.62} \left(M_0,\ \begin{array}{l} t_1 = \mathbf{0.62}, \\ t_3 = \mathbf{0.62} \end{array}\right) \xrightarrow{t_1} \left(\mathbf{M_1},\ \begin{array}{l} t_1 = 0.62, \\ \mathbf{t_2} = \mathbf{0} \end{array}\right)$$

About newly enabled transitions



Fire t_1

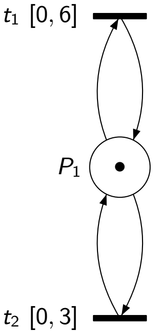
About newly enabled transitions



Fire t_1

t_1 and t_2 are not enabled by $M - {}^\bullet t_1$

About newly enabled transitions

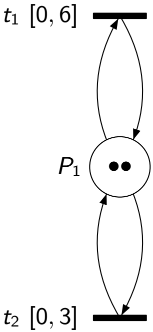


Fire t_1

t_1 and t_2 are not enabled by $M - {}^\bullet t_1$

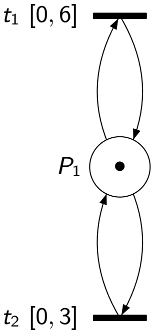
t_1 and t_2 are **newly** enabled

About newly enabled transitions



Fire t_1

About newly enabled transitions



Fire t_1

t_1 and t_2 are enabled by $M - {}^\bullet t_1$ but t_1 is the fired transition

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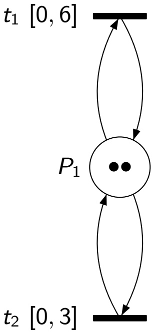
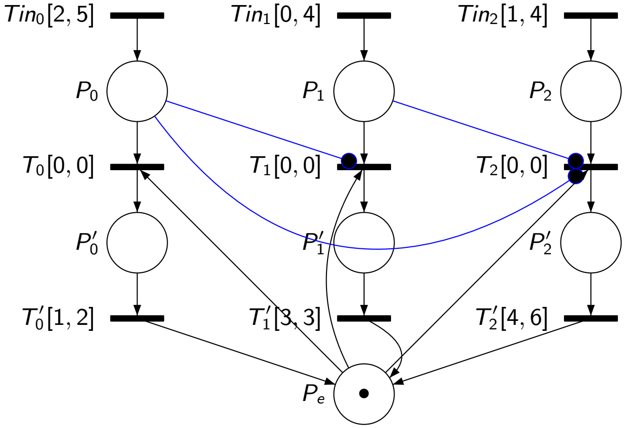


Fire t_1

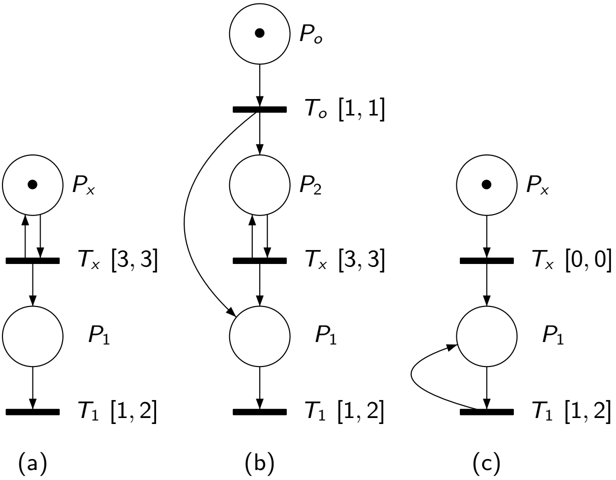
t_1 and t_2 are enabled by $M - {}^\bullet t_1$ but t_1 is the fired transition

t_2 **remains** enabled, t_1 is **newly** enabled

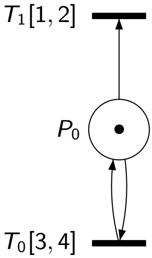
Time Petri Net Example: Fixed Priority Scheduling (non-preemptive)



TPN Example: Task Activation



Read Arcs and Time



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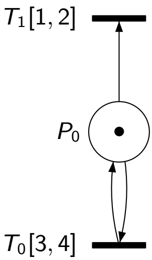

Read should not be destructive:

$$\left(M_0, \begin{array}{l} T_0 = 0, \\ T_1 = 0 \end{array} \right)$$

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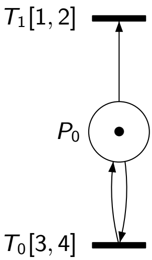

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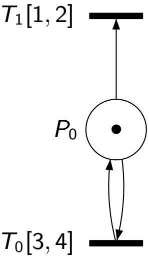

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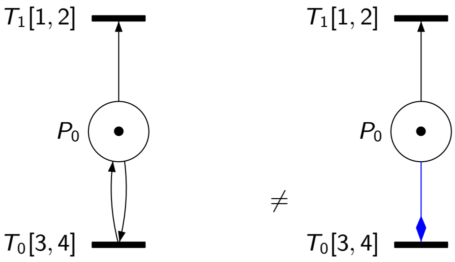

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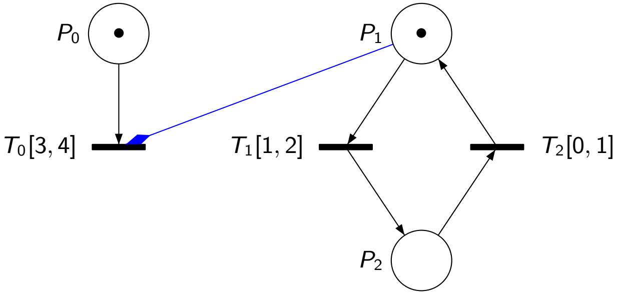
From Read Arcs To Activator Arcs



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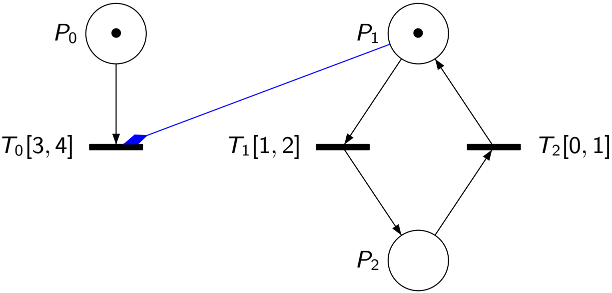
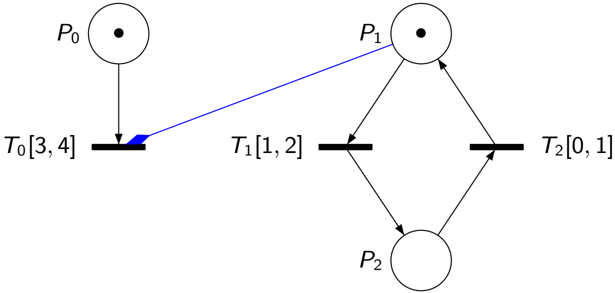


Memory / No Memory:

$$\left(M_0, \begin{array}{l} T_0 = 0, \\ T_1 = 0 \end{array} \right)$$

From Read Arcs To Activator Arcs



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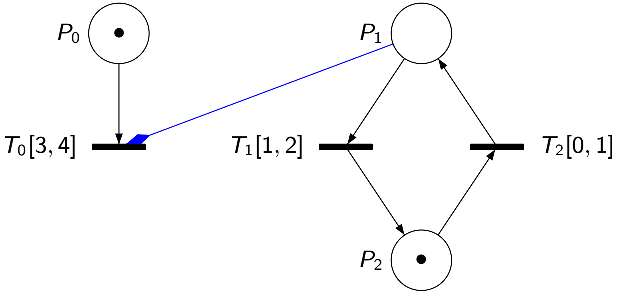
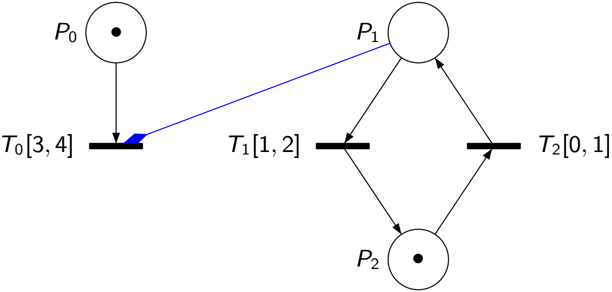


Memory / No Memory:

$$\left(M_0,\ \begin{array}{l} T_0 = 0, \\ T_1 = 0 \end{array}\right) \xrightarrow{1.62} \left(M_0,\ \begin{array}{l} T_0 = 1.62, \\ T_1 = 1.62 \end{array}\right) \xrightarrow{T_1} (M_1, T_2 = 0)$$

From Read Arcs To Activator Arcs

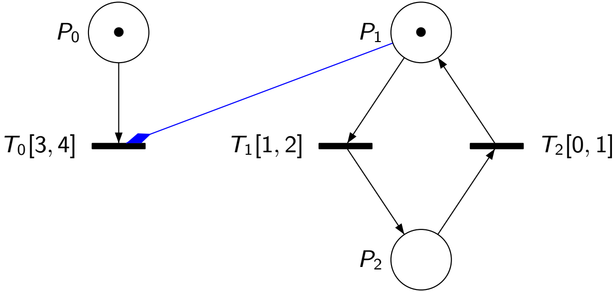


Memory / No Memory:

$$\left(M_0, \begin{array}{l} T_0 = 0, \\ T_1 = 0 \end{array}\right) \xrightarrow{1.62} \left(M_0, \begin{array}{l} T_0 = 1.62, \\ T_1 = 1.62 \end{array}\right) \xrightarrow{T_1} (M_1, T_2 = 0)$$

$$\xrightarrow{0.2} (M_1, T_2 = 0.2)$$

From Read Arcs To Activator Arcs



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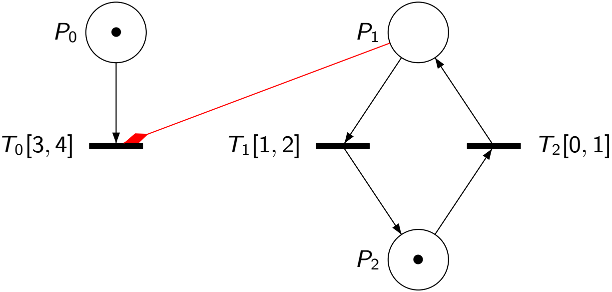
$$\xrightarrow{0.2} (M_1, T_2 = 0.2) \xrightarrow{T_2} \left(M_0, \begin{array}{l} T_0 = \mathbf{0}, \\ T_1 = 0 \end{array}\right)$$

From Read Arcs To Activator Arcs

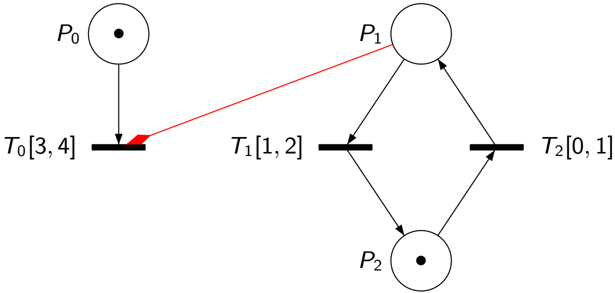


Memory / No Memory:

$$\left(M_0, \begin{array}{l} T_0 = 0, \\ T_1 = 0 \end{array}\right) \xrightarrow{1.62} \left(M_0, \begin{array}{l} T_0 = 1.62, \\ T_1 = 1.62 \end{array}\right) \xrightarrow{T_1} \left(M_1, \begin{array}{l} T_0 = 1.62, \\ T_2 = 0 \end{array}\right)$$

$$\xrightarrow{0.2} (M_1, T_2 = 0.2) \xrightarrow{T_2} \left(M_0, \begin{array}{l} T_0 = \mathbf{0}, \\ T_1 = 0 \end{array}\right) \xrightarrow{0.31} \left(M_0, \begin{array}{l} T_0 = 0.31, \\ T_1 = 0.31 \end{array}\right)$$

From Read Arcs To Activator Arcs



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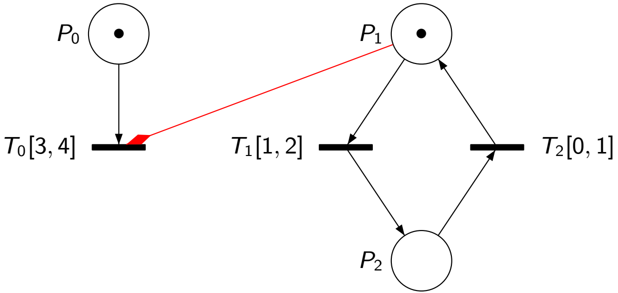

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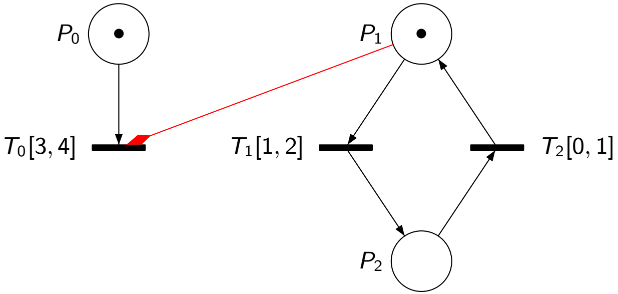

Memory / No Memory:

$$\left(M_0, \begin{array}{l} T_0 = 0, \\ T_1 = 0 \end{array}\right) \xrightarrow{1.62} \left(M_0, \begin{array}{l} T_0 = 1.62, \\ T_1 = 1.62 \end{array}\right) \xrightarrow{T_1} \left(M_1, \begin{array}{l} T_0 = 1.62, \\ T_2 = 0 \end{array}\right)$$

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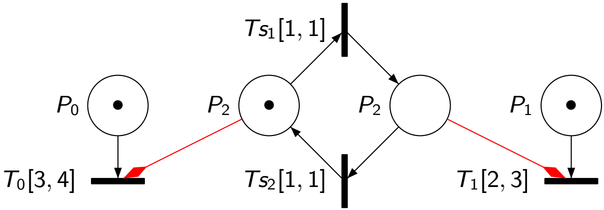
TPN with Stopwatches Example: Round-Robin Scheduling



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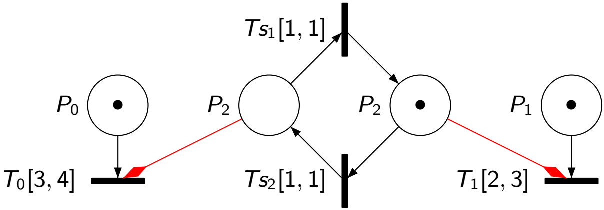

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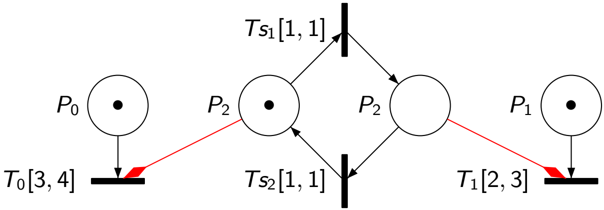
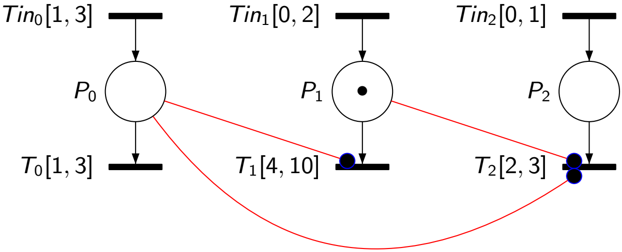
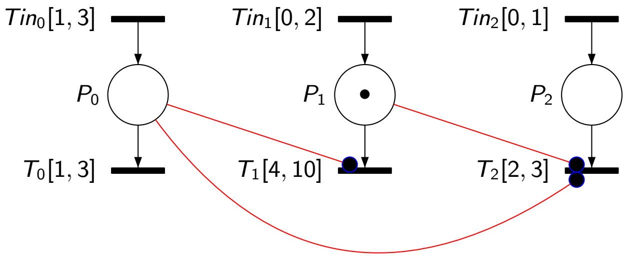


Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



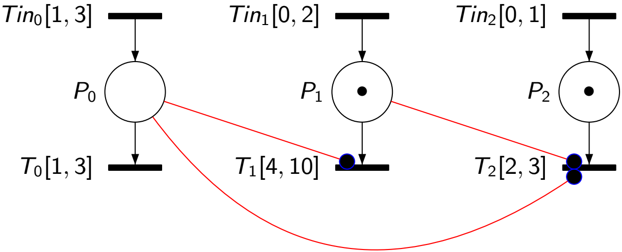
$$T_0 =? \quad T_1 = 0 \quad T_2 =?$$

Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



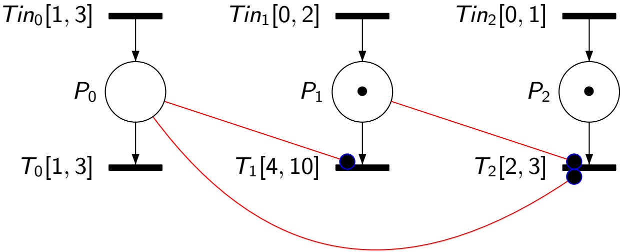
$T_0 = ?\quad T_1 = \mathbf{0.22}\quad T_2 = ?$

Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



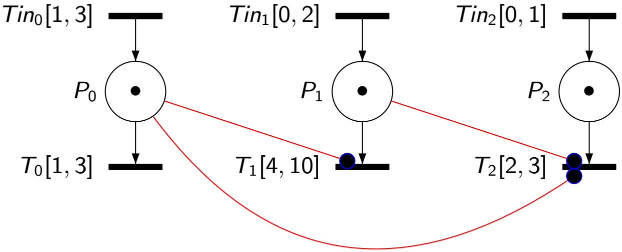
$$T_0 = ? \quad T_1 = 0.22 \quad T_2 = \mathbf{0}$$

Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



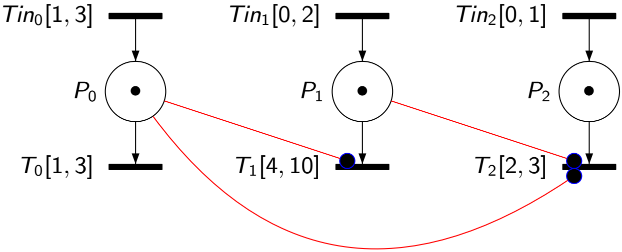
$$T_0 = ? \quad T_1 = \mathbf{1.6} \quad T_2 = 0$$

Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



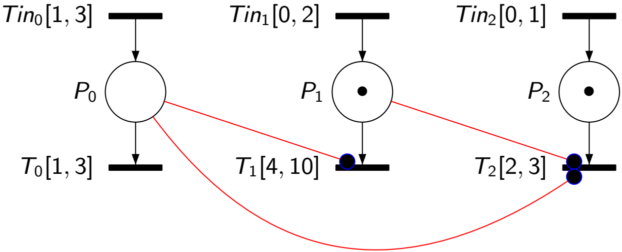
$$T_0 = \mathbf{0} \quad T_1 = 1.6 \quad T_2 = 0$$

Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



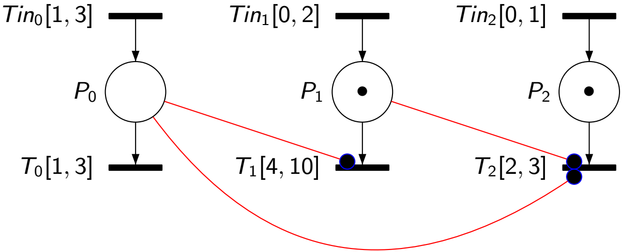
$T_0 = \mathbf{1.5} \quad T_1 = 1.6 \quad T_2 = 0$

Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



$$T_0 =? \quad T_1 = 1.6 \quad T_2 = 0$$

Time Inhibitor Arcs: Fixed Priority Scheduling (Preemptive)



$T_0 = ?\quad T_1 = \mathbf{1.8}\quad T_2 = 0$

The price of expressiveness

The **reachability** problem:

	General case	Bounded
Petri Nets	decidable	decidable
Petri Nets w/ Inhibitor	undecidable[8]	decidable
Time Petri Nets	undecidable [9]	decidable [1]
Stopwatch Time Petri Nets	undecidable [2]	undecidable[2]

Scheduling Time Petri Nets

- A model in the class of **Time Petri Nets w/ Stopwatches**;

Scheduling Time Petri Nets

- A model in the class of **Time Petri Nets w/ Stopwatches**;
- Dedicated to the modelling of **preemptive scheduling** policies;

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- Tasks are given a **processor** and **priority** (or deadline or . . .);

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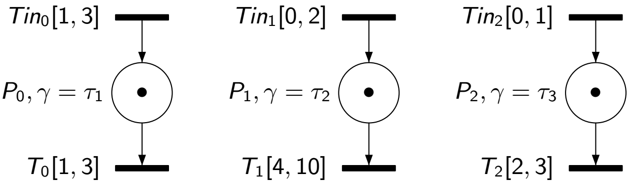
Scheduling Time Petri Nets

- A model in the class of **Time Petri Nets w/ Stopwatches**;
- Dedicated to the modelling of **preemptive scheduling** policies;
- Places and transitions are associated with **tasks**;
- Tasks are given a **processor** and **priority** (or deadline or . . .);
- Processors are given a **scheduling policy**.
- With the state of the net of **progress rate for each transition** is computed at each change of marking.

Example: Fixed Priority Scehduling (Preemptive)

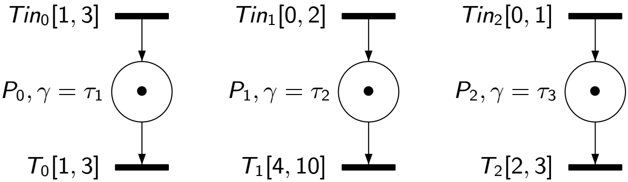


$Proc(\tau_1) = 1$
$Prio(\tau_1) = 3$

$Proc(\tau_2) = 1$
$Prio(\tau_2) = 2$

$Proc(\tau_3) = 1$
$Prio(\tau_3) = 1$

Example: Fixed Priority Scehduling (Preemptive)



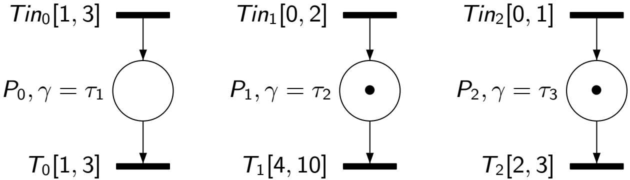
$Proc(\tau_1) = 1$ $Prio(\tau_1) = 3$

$Proc(\tau_2) = 1$ $Prio(\tau_2) = 2$

$Proc(\tau_3) = 1$ $Prio(\tau_3) = 1$

$$Flow(T_0) = \mathbf{1}, Flow(T_1) = 0, Flow(T_2) = 0$$

Example: Fixed Priority Scehduling (Preemptive)



$Proc(\tau_1) = 1$
$Prio(\tau_1) = 3$

$Proc(\tau_2) = 1$
$Prio(\tau_2) = 2$

$Proc(\tau_3) = 1$
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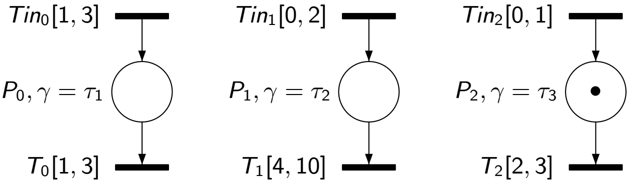
$$Flow(T_0) =?, Flow(T_1) = \mathbf{1}, Flow(T_2) = 0$$

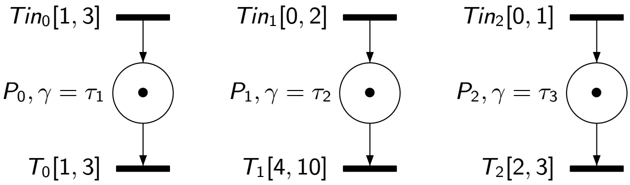
Example: Fixed Priority Scehduling (Preemptive)



$Proc(\tau_1) = 1$	$Proc(\tau_2) = 1$	$Proc(\tau_3) = 1$
$Prio(\tau_1) = 3$	$Prio(\tau_2) = 2$	$Prio(\tau_3) = 1$

$$Flow(T_0) = ?, Flow(T_1) = ?, Flow(T_2) = \mathbf{1}$$

Example: Round-Robin

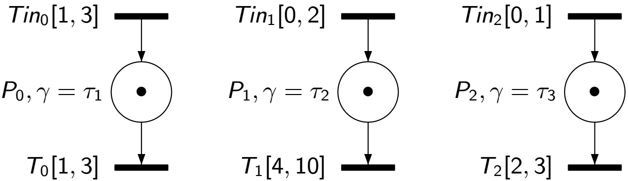


$Proc(\tau_1) = 1$
$Prio(\tau_1) = 1$

$Proc(\tau_2) = 1$
$Prio(\tau_2) = 1$

$Proc(\tau_3) = 1$
$Prio(\tau_3) = 1$

Example: Round-Robin

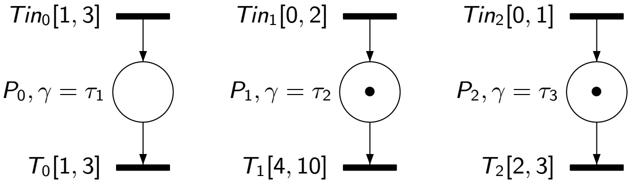


$Proc(\tau_1) = 1$ $Proc(\tau_2) = 1$ $Proc(\tau_3) = 1$

$Prio(\tau_1) = 1$ $Prio(\tau_2) = 1$ $Prio(\tau_3) = 1$

$$Flow(T_0) = \mathbf{\frac{1}{3}}, Flow(T_1) = \mathbf{\frac{1}{3}}, Flow(T_2) = \mathbf{\frac{1}{3}}$$

Example: Round-Robin

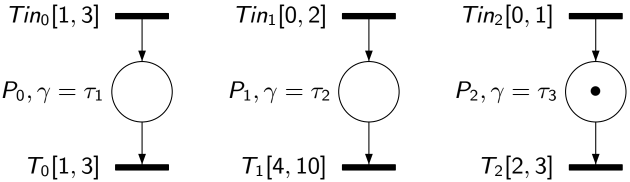


$Proc(\tau_1) = 1$ $Proc(\tau_2) = 1$ $Proc(\tau_3) = 1$

$Prio(\tau_1) = 1$ $Prio(\tau_2) = 1$ $Prio(\tau_3) = 1$

$$Flow(T_0) =?, Flow(T_1) = \mathbf{\frac{1}{2}}, Flow(T_2) = \mathbf{\frac{1}{2}}$$

Example: Round-Robin



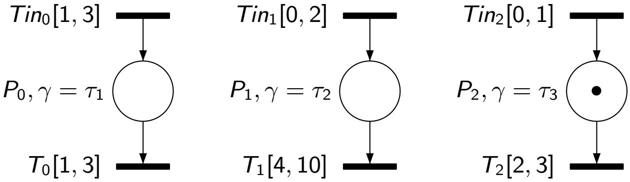
$Proc(\tau_1) = 1$ $Prio(\tau_1) = 1$

$Proc(\tau_2) = 1$ $Prio(\tau_2) = 1$

$Proc(\tau_3) = 1$ $Prio(\tau_3) = 1$

$$Flow(T_0) = ?, Flow(T_1) = ?, Flow(T_2) = \mathbf{1}$$

Example: Round-Robin

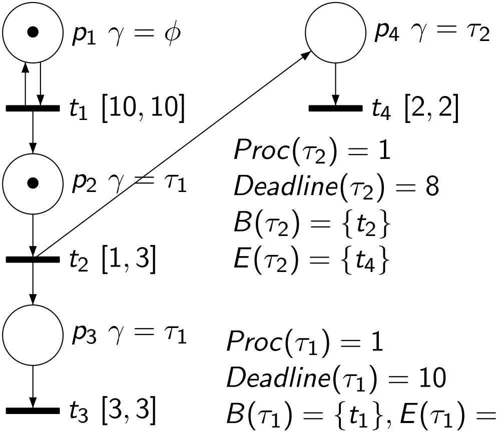


$Proc(\tau_1) = 1$ $Proc(\tau_2) = 1$ $Proc(\tau_3) = 1$

$Prio(\tau_1) = 1$ $Prio(\tau_2) = 1$ $Prio(\tau_3) = 1$

A **fluid** approach: minimize the number of discrete changes.

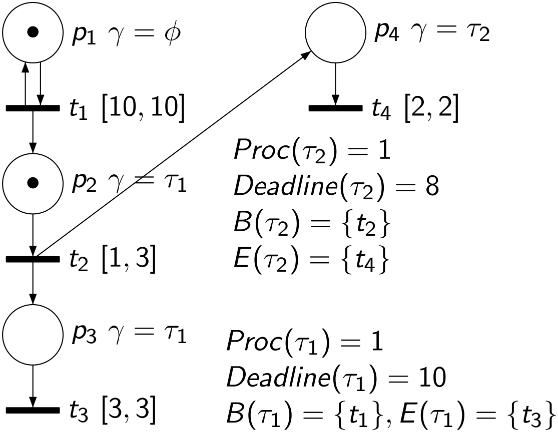
Example: Earliest Deadline First



$Proc(\tau_2) = 1$
$Deadline(\tau_2) = 8$
$B(\tau_2) = \{t_2\}$
$E(\tau_2) = \{t_4\}$

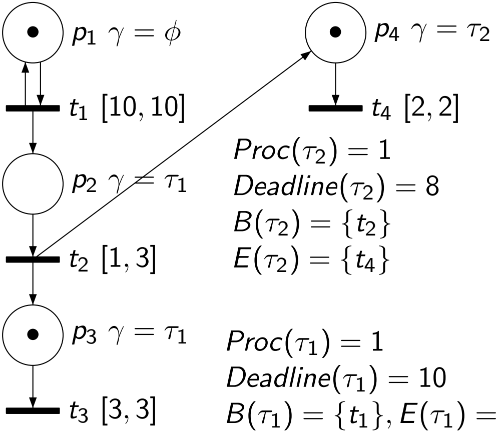
$Proc(\tau_1) = 1$
$Deadline(\tau_1) = 10$
$B(\tau_1) = \{t_1\}, E(\tau_1) = \{t_3\}$

Example: Earliest Deadline First



$Flow(t_1) = 1 \quad Flow(t_2) = 1 \quad Flow(t_3) =? \quad Flow(t_4) =?$

Example: Earliest Deadline First

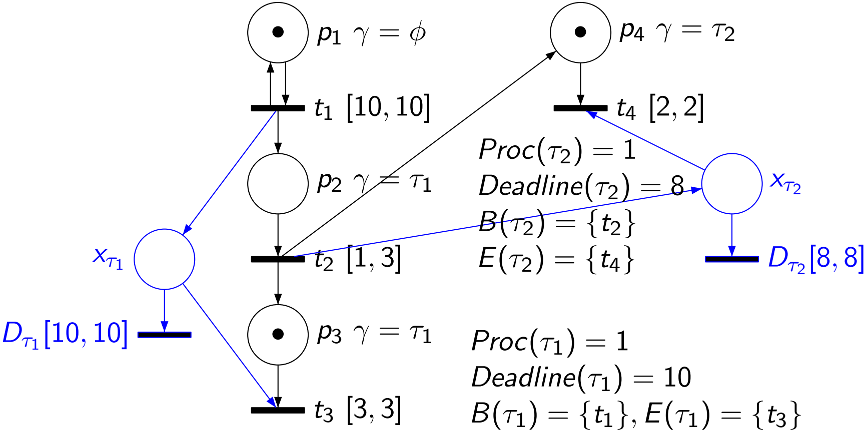


$Proc(\tau_2) = 1$
$Deadline(\tau_2) = 8$
$B(\tau_2) = \{t_2\}$
$E(\tau_2) = \{t_4\}$

$Proc(\tau_1) = 1$
$Deadline(\tau_1) = 10$
$B(\tau_1) = \{t_1\}, E(\tau_1) = \{t_3\}$

if t_2 was fired **before** 2
$Flow(t_1) = 1$ $Flow(t_2) = ?$ $Flow(t_3) = 0$ $Flow(t_4) = \mathbf{1}$
if t_2 was fired **after** 2
$Flow(t_1) = 1$ $Flow(t_2) = ?$ $Flow(t_3) = \mathbf{1}$ $Flow(t_4) = 0$

Example: Earliest Deadline First



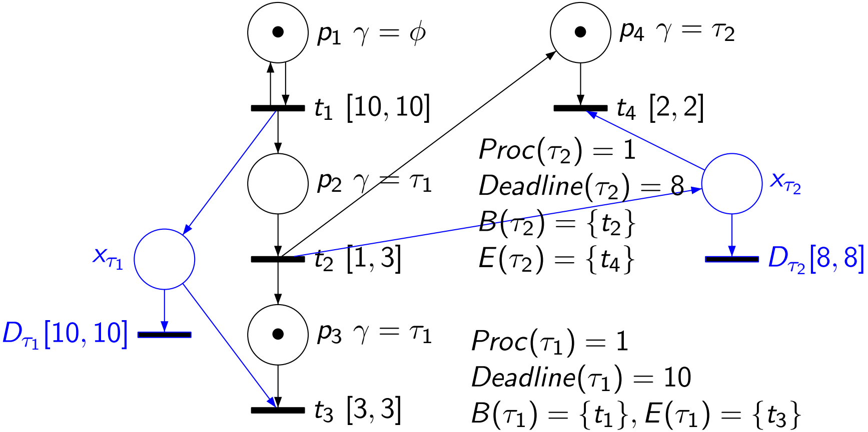
if t_2 was fired **before** 2

$Flow(t_1) = 1$ $\quad Flow(t_2) = ?$ $\quad Flow(t_3) = 0$ $\quad Flow(t_4) = \mathbf{1}$

if t_2 was fired **after** 2

$Flow(t_1) = 1$ $\quad Flow(t_2) = ?$ $\quad Flow(t_3) = \mathbf{1}$ $\quad Flow(t_4) = 0$

Example: Earliest Deadline First



if $\mathbf{D_{\tau_2}} \leq \mathbf{D_{\tau_1}}$

$Flow(t_1) = 1 \quad Flow(t_2) = ? \quad Flow(t_3) = 0 \quad Flow(t_4) = \mathbf{1}$

if $\mathbf{D_{\tau_1}} < \mathbf{D_{\tau_2}}$

$Flow(t_1) = 1 \quad Flow(t_2) = ? \quad Flow(t_3) = \mathbf{1} \quad Flow(t_4) = 0$

Plan I

Abstractions

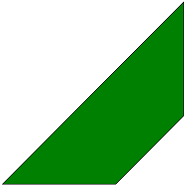
- **Infinite** state-space ⇒ Abstractions
- TPNs: Zone-based simulation graph [5]
- TPNs: **State class graph** [1]
- TPNs w/ stopwatches (IHTPNs,...): **State class graph** [10, 11, 3]

Basic Algorithm

begin
  $Passed = \emptyset$
  $Waiting = \{C_0\}$
  while $Waiting \neq \emptyset$
    $C = \text{pop}(Waiting)$
    $Passed = Passed \cup C$
    for t firable from C
      $C' = \text{AbstractSuccessor}(\mathbf{C}, \mathbf{t})$
      if $C' \notin Passed$
        $Waiting = Waiting \cup C'$
      end if
    end for
  end while
end

State Class

$$C = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \right.$$



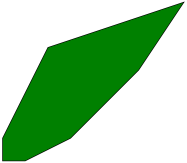
$$\left. \right\}$$

TPNs: Zone (encoded by a Difference Bound Matrix (DBM) $[d_{ij}]_{i,j \in [0..n]}$):

$$\begin{cases} -d_{0i} \leq \theta_i - \mathbf{0} \leq d_{i0}, \\ \theta_i - \theta_j \leq d_{ij} \end{cases}$$

State Class

$$C = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \right.$$

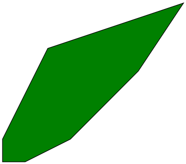


$$\left. \right\}$$

SwTPNs: General polyhedron: $A\bar{\Theta} \leq B$

Over-approximation

$$C = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \right.$$

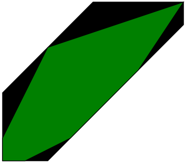


$$\left. \right\}$$

Over-approximation using the **smallest englobing** zone

Over-approximation

$$C = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \right.$$



$$\left. \right\}$$

Over-approximation using the **smallest englobing** zone

Computing the state class graph (normal)

Let $C = (M, D)$ and $D = (A.\Theta \leq B)$. We fire t_f.

- $M' = M - {}^\bullet t_f + t_f{}^\bullet$
- D' is computed by:
  - for all enabled transitions t_i s.t. $Flow(t_i) \neq 0$, constrain by $\theta_f \leq \theta_i$
  - for all enabled transitions t_i s.t. $Flow(t_i) \neq 0$, $\theta_i' = \theta_i - \theta_f$
  - eliminate variables for **disabled** transitions (e.g. using Fourier-Motzkin method [4])
  - add new variables for **newly** enabled transitions t_i:

$$\alpha(t_i) \leq \theta_i \leq \beta(t_i)$$

Computing the state class graph (round-robin)

Let $C = (M, D)$ and $D = (A.\Theta \leq B)$. We fire t_f.

- $M' = M - {}^\bullet t_f + t_f{}^\bullet$
- D' is computed by:
  - for all enabled transitions t_i s.t. $Flow(t_i) \neq 0$, constrain by $\theta_f \leq \theta_i$
  - for all enabled transitions t_i s.t. $Flow(t_i) \neq 0$, $\theta'_i = \theta_i - \frac{\mathbf{Flow(t_j)}}{\mathbf{Flow(t_f)}}\theta_f$
  - eliminate variables for **disabled** transitions (e.g. using Fourier-Motzkin method [4])
  - add new variables for **newly** enabled transitions t_i:

$$\alpha(t_i) \leq \theta_i \leq \beta(t_i)$$

Computing the state class graph (earliest deadline first)

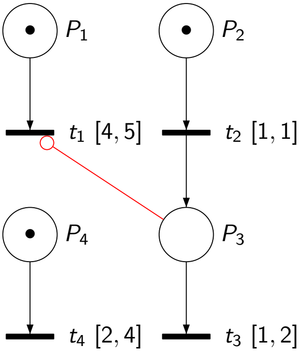
Let $C = (M, D)$ and $D = (A.\Theta \leq B)$. We fire t_f.

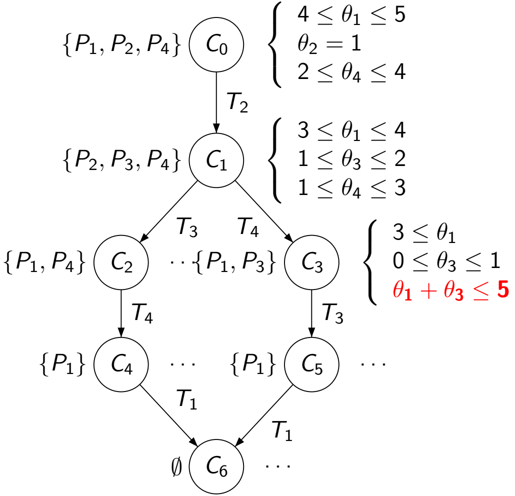
- $M' = M - {}^\bullet t_f + t_f{}^\bullet$
- D' is computed by:
  - for all enabled transitions t_i s.t. $Flow(t_i) \neq 0$, constrain by $\theta_f \leq \theta_i$
  - for all enabled transitions t_i s.t. $Flow(t_i) \neq 0$, $\theta'_i = \theta_i - \theta_f$
  - eliminate variables for **disabled** transitions (e.g. using Fourier-Motzkin method [4])
  - add new variables for **newly** enabled transitions t_i:

$$\alpha(t_i) \leq \theta_i \leq \beta(t_i)$$

  - **partition** D with $D_\tau \leq D_{\tau'}$ and $D_\tau > D_{\tau'}$

Example


P_1
P_2
t_1 $[4, 5]$
t_2 $[1, 1]$
P_4
P_3
t_4 $[2, 4]$
t_3 $[1, 2]$


$\{P_1, P_2, P_4\}$ C_0
$\begin{cases} 4 \leq \theta_1 \leq 5 \\ \theta_2 = 1 \\ 2 \leq \theta_4 \leq 4 \end{cases}$
T_2
$\{P_2, P_3, P_4\}$ C_1
$\begin{cases} 3 \leq \theta_1 \leq 4 \\ 1 \leq \theta_3 \leq 2 \\ 1 \leq \theta_4 \leq 3 \end{cases}$
T_3
T_4
$\{P_1, P_4\}$ C_2
$\cdots$ $\{P_1, P_3\}$ C_3
$\begin{cases} 3 \leq \theta_1 \\ 0 \leq \theta_3 \leq 1 \\ \boldsymbol{\theta_1 + \theta_3 \leq 5} \end{cases}$
T_4
T_3
$\{P_1\}$ C_4 $\cdots$
$\{P_1\}$ C_5 $\cdots$
T_1
T_1
$\emptyset$ C_6 $\cdots$

The Way of the Middle (1/2)

For IHTPNs:

- The polyhedron in the **initial** state class is **always a zone**.
- The successor of a non-zone polyhedron might be a zone
- The successor of a zone might not be a zone

The Way of the Middle (1/2)

For IHTPNs:

- The polyhedron in the **initial** state class is **always a zone**.
- The successor of a non-zone polyhedron might be a zone
- The successor of a zone might not be a zone

We want to start to compute with zones and fall back to general polyhedra when needed (and return to zones asap).

The Way of the Middle (1/2)

For IHTPNs:

- The polyhedron in the **initial** state class is **always a zone**.
- The successor of a non-zone polyhedron might be a zone (**easy to check**: $O(n^2)$)
- The successor of a zone might not be a zone (**not so easy to check**)

We want to start to compute with zones and fall back to general polyhedra when needed (and return to zones asap).

Abstractions: The Way of the Middle (2/2)

$D = [d_{ij}]_{i,j \in [0..n]}$ and $(M', D') = AbstractSuccessor((M, D), t_f)$.

D' is **not a zone** iff there are at least three enabled transitions t_i, t_j, t_k in D such that:

1. t_i, t_j, t_k are not disabled when firing t_f;
2. $Flow(t_i) \neq 0$ and $Flow(t_k) \neq 0$;
3. $Flow(t_j) = 0$;
4. $d_{j0} + d_{ki} > d_{k0} + d_{ji}$ or $d_{0j} - d_{ik} < d_{0k} - d_{ij}$

Abstractions: The Way of the Middle (2/2)

$D = [d_{ij}]_{i,j \in [0..n]}$ and $(M', D') = AbstractSuccessor((M, D), t_f)$.

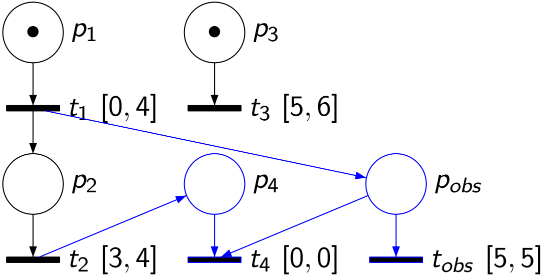
D' is **not a zone** iff there are at least three enabled transitions t_i, t_j, t_k in D such that:

1. t_i, t_j, t_k are not disabled when firing t_f;
2. $Flow(t_i) \neq 0$ and $Flow(t_k) \neq 0$;
3. $Flow(t_j) = 0$;
4. $d_{j0} + d_{ki} > d_{k0} + d_{ji}$ or $d_{0j} - d_{ik} < d_{0k} - d_{ij}$

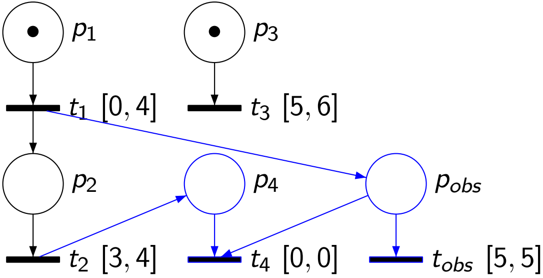
Complexity: $O(n^3)$

Plan I

Observing durations

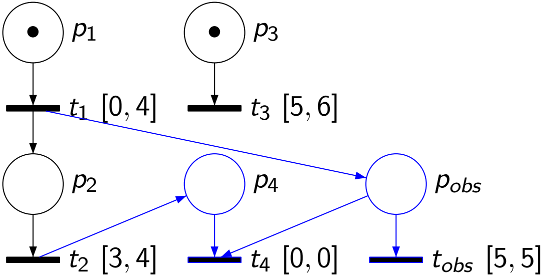


Observing durations



There can be ≥ 5 t.u. between the firings of t_1 and t_2 iff t_{obs} is fired.

Observing durations



$\boxed{\text{the max sojourn time of the token is } \max_{(M,D)\in \text{Classes}}\{5 - \min(D_{|t_{obs}})\}.}$

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- **Warning**: It cannot express properties on stopwatches (response times but not cumulated execution durations).

Plan I

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- Some **tools** exist including ROMEO (IRCCyN, Nantes) and TINA (LAAS, Toulouse).

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