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A New Method for the Dynamic Formulation of Parallel Manipulators

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This paper proposes a new method for the complete forward and inverse dynamics models of parallel robot. These methods have been applied to a six d.o.f. Gough-Stewart manipulator. The inverse dynamic model, calculates the joint torques/forces as a function of positions, velocities and accelerations of the platform operational coordinates ($\mathbf{\Gamma = f(X, \upsilon, \dot{\upsilon})}$). The direct dynamic model, gives the joint accelerations as a function of joint positions, velocities of the platform operational coordinates and torques/forces ($\mathbf{\dot{\upsilon} = f(X, \upsilon, \Gamma)}$).

The Gough-Stewart manipulator is composed of a moving platform connected to a fixed base by six extendable legs. The two extremities of each leg are fitted with a universal joint at the base and a spherical joint at the platform (fig. 1) :

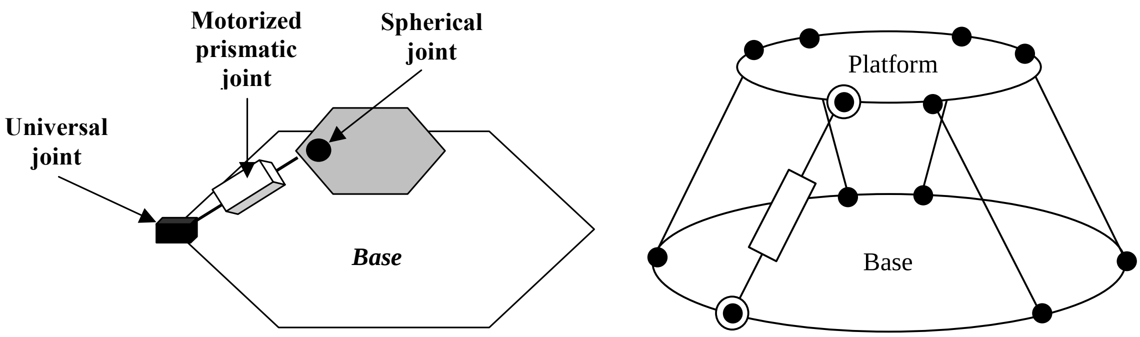

Figure 1. : 6 d.o.f Gough-Stewart manipulator

The robot has a complex closed structure with five spatial closed loops. In the classical method the closed loop are opened to build a tree equivalent structure. The dynamic model of the robot can be obtained from the dynamic model of the tree structure and the Jacobian matrix of tree's variables with respect to the active joints. This Jacobian matrix could be obtained from geometric or kinematic constraint equations of the loops.

In the proposed method, the platform is supposed isolated to obtain a minimal tree equivalent structure composed of the base and the six legs (Fig. 2). Because of the type of joints used (spherical joint) only pure force $\mathbf{f_i}$ $\left(f_i = [f_{xi} \quad f_{xi} \quad f_{xi}]^T\right)$ can exist between leg i (i = 1 to 6) and the platform (fig. 3).

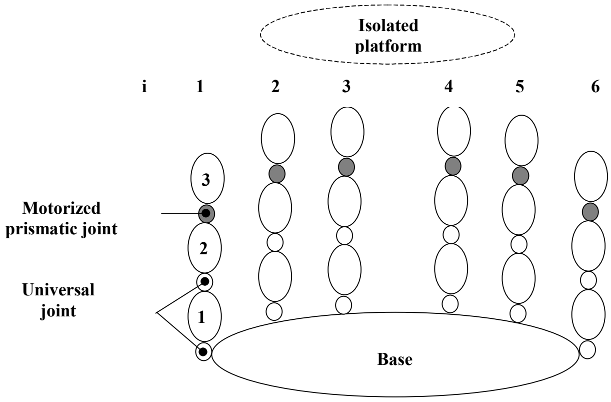

Figure 2. : Minimal tree equivalent structure

Inverse dynamic model :

In this case, there are 24 unknowns consisting of the 18 components of the six forces $\mathbf{f_i}$ and the six forces of the motorized joints Γ_{3i}. The twelve torques from the two passive joints of the six universal joints are nulls (fig. 3). We have 24 equations consisting of the 18 dynamics equations of the legs and the six Newton-Euler equations of the platform. These systems of equations will be solved sequentially as follows :

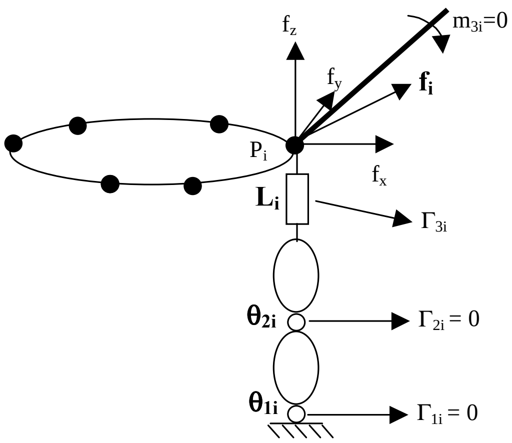

Figure 3. : Description of one leg

The dynamic model of one leg gives the relation between the force $\mathbf{f_i}$ (i = 1 to 6) and the forces/torques Γ_{1i} Γ_{2i} Γ_{3i} (i = 1 to 6). The elements f_{xi} and f_{yi} are function respectively of Γ_{2i} and Γ_{1i}, they could be evaluated for any motion of platform. The element f_{zi} is a function of Γ_{3i}.

Then, with the equilibrium equations of the platform (six Newton-Euler equations), the values of the elements f_{zi} could be found by resolving a linear system of six equation and six unknowns. Finally, the six forces Γ_{3i} (i = 1 to 6) of the six motorized joints could be deducted.

Direct dynamic model :

For the direct dynamic model, the joint accelerations could be expressed as functions of the platform operational coordinates accelerations, from the second order kinematic model of each leg. Next, the inverse dynamic model of each leg is used to express the forces $\mathbf{f}_\mathbf{i}$ (i = 1 to 6) as functions of the platform operational coordinates accelerations.

Then, with the equilibrium equations of the platform the values of the platform operational coordinates accelerations could be found by resolving a linear system of six equations and six unknowns. Finally, the joints accelerations $\ddot{q}_{1i}, \ddot{q}_{2i}, \ddot{q}_{3i}$ (i = 1 to 6) of the six legs could be deducted.