Diagnosis of nonlinear system using a reduced order fault observer:Application to a bioreactor

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Abstract. Nonlinear system diagnosis problem in a bioreactor process is addressed. We consider the algebraic observability concept of the variable which models the failure presence for the solvability of the problem. the key ingredient is the construction of a reduced order fault observer to estimate the fault variable.

Keywords: Diagnosis problem, diagnosability condition, reduced order fault observer, bioreactor.

1 Introduction

A key question in bioprocess control is how to detect, identify, estimate, and accommodate faults in the process. The term *fault* means process degradation or degradation of the equipment performance caused by some change in the physical characteristics of the process, the input process or the ambient conditions. In the general case, there are many different approaches to solve the diagnosis problem (fault detection and identification problem) in nonlinear systems. In this sense, there are several authors who have proposed solutions to the Fault Detection and Identification Problem in a system class ([1], [2], [4], [5]). For example in [1] and [2] it has been considered an approach to diagnosis problem which consists of translating the solvability of the problem in terms of the algebraic observability of the variable which models failure presence, and which is usually called the *fault*. The work is conceived in the language of differential algebra which allows to define algebraic observability concept. In [3],[6],[4] the methodologies employed for the observer design only include full order observers which do not include uncertainty estimation.

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In this paper, we consider the fault variable dynamics as uncertainty and the using of a full order observer is not necessary because we do not require of redundant information. So that, we construct a reduced order uncertainty observer using differential algebraic techniques applied to the fault estimation in the diagnosis problem. The methodology consists in the following steps: First, from nominal system we take a reduced system which include the fault variable, in the step second, the dynamics of the fault is considered as an uncertainty term, and we construct a reduced order uncertainty observer (see [9]) which solve the diagnosis problem. We can detect the faults, by examining the dynamics of the faults. In absence of faults, the fault dynamics goes to zero (normal operation) while if a fault occurs, that is to say, whether the fault shows a distinguishable nonzero value, the observer gives a good estimation of the chosen fault function (not normal operation). Here, we have considered the concept of diagnosability as is given in [1] and [2].

In this work, the solvability of diagnosis problem that we propose is based upon the following points: 1) the availability of fault estimates fully answers the three aspects of diagnosability: detection, identification and estimation of faults. 2) The ability to estimate the fault variable is readily seen as the observability of the fault with respect to the data consisting of time histories of u and y. That is to say, a system is diagnosable if its fault variable is observable with respect to u and y. 3) For each fault component, we can obtain a relation satisfied by this fault component and the derivatives of the data (input, output), and called the diagnosability condition of the given fault component.

The paper is organized as follows: in section 2 we introduce some basic definitions on system diagnosability in differential algebraic framework. Statement of the problem and the reduced order uncertainty observer synthesis are described in section 3. The application of the synthesis algorithm to bioreactors in the diagnosis problem is given in section 4. Section 5 is dedicated to show some numerical results. Finally, in section 6 we will close the paper with some concluding remarks.

2 Basic definitions

We start introducing some basic definitions and notation in the study of diagnosis problem (see [1]).

Definition 1 *Differential field extension L/k is given by two differential fields k and L, such that: i) k is a subfield of L, ii) the derivation of k is the restriction to k of the derivation of L.*

Definition 2 *A dynamics is a finitely generated differential algebraic extension $G/k\langle u\rangle$ $(G = k\langle u, \xi\rangle, \xi \in G)$. Any element of G satisfies an algebraic differential equation with coefficients which are rational functions over k in the components of u and a finite number of their time derivatives.*

Definition 3 *Let a subset* $\{u, y\}$ *of* G *in a dynamics* $G/k\langle u\rangle$. *An element in* G *is said to be algebraically observable with respect to* $\{u, y\}$ *if it is algebraic over* $k\langle u, y\rangle$. *Therefore a state* x *is said to be algebraically observable if, and only if, it is algebraically observable with respect to* $\{u, y\}$. *A dynamics* $G/k\langle u\rangle$ *with output* y *in* G *is said to be algebraically observable if, and only if, any state has this property.*

The algebraic observability means that the differential field extension $G/k\langle u, y\rangle$ is algebraic, i.e., the whole differential information is contained in $k\langle u, y\rangle$.

Let nonlinear system be given by:

$$\begin{aligned} \dot{x}(t) &= A(x, f, u) \\ y(t) &= h(x, u) \end{aligned} \quad (1)$$

where $x = (x_1, ..., x_{n_1})^T \in R^{n_1}$ is a state vector, $f \in R^{n_2}$ is a fault vector, $u = (u_1, ..., u_m)^T \in R^m$ is an input vector, $y \in R^p$ is the output measured vector, A and h are assumed to be analytical vector functions.

Definition 4 *A system like (1) is said to be diagnosable if it is possible to estimate the fault* f *from the system equations and the time histories of the data* u *and* y*, i.e., is diagnosable if* f *is observable with respect to* u *and* y.

Definition 5 *The fault* f *is observable with respect to* u *and* y *if each component* f_i *is algebraic over the differential field extension of* k *generated by the data* u, y.

A new concept is considered in order to define an algebraically observable fault condition.

Definition 6 *An element* f *in* G *is said to be an algebraically observable if* f *satisfies a differential algebraic equation with coefficients over* $k\langle u, y\rangle$.

The algebraic observability notion requires that each fault component be able to be written as a solution of a polynomial equation

$$H_i(f_i, u, \dot{u}, ..., y, \dot{y}, ...) = 0 \quad (2)$$

in f_i, and finitely many time derivatives of u and y, with coefficients in k.

Definition 7 *A system (1) is said to be uniquely diagnosable if it is diagnosable and the diagnosability conditions (2) have unique solutions* f_i *in terms of* u, y *and their time derivatives.*

Definition 8 *A rationally diagnosable system is one for which the diagnosability conditions (2) are linear in the* f_i*'s. A differential algebraic system (1) is rationally diagnosable if and only if it is diagnosable and its undisturbed defining differential field extension* $k\langle u, f, x, y\rangle$, *is equal to its external behavior differential field extension* $k\langle u, y\rangle$.

If system (1) is rationally diagnosable, then, for each fault component f_i, the diagnosability condition (2) reduces to

$$f_i = \frac{h_i(u, y)}{q_i(u, y)}$$

where h_i and q_i are differential polynomials with coefficients in k.

Linear systems are rationally diagnosable if and only if they are diagnosable. For nonlinear systems the situation is different, although a system being rationally diagnosable, there exists outputs which make the system becomes non diagnosable, i.e., there are *singular* observation data for the diagnosability of the system.

If a system is not diagnosable in the algebraic sense, but we still can estimate th fault f given the stability of its dynamics in terms of u, y and their time derivatives, then we would say here that the f is *detectable* with respect to u and y.

Remark 1 *[1], [2] The general result is that a system which is observable, is diagnosable if and only if its fault variable f is observable with respect to u, y and x.*

Remark 2 *[1], [2] Assume that system (1) is with no control, then it is diagnosable only if it has as many measurements as fault variables.*

3 Statement of the problem

We consider the nonlinear systems given by (1), the fault vector f is unknown and we can assimilate it as a state with uncertain dynamics, then to estimate it we can extend the state vector deal with the unknown fault vector [8] and the new system is given by:

$$\begin{array}{l} \dot{x}(t) = A(x, f, u) \\ \dot{f}(t) = \Omega(x, f, u) \\ y(t) = h(x, u) \end{array} \qquad (3)$$

Where $\Omega(x, f, u)$ is considered as an uncertain function which is bounded.

We suppose that the system (3) is universally observable in sense of definition 3, with external behaviour given by equations of the form:

$$y_{\bar{k}}^{(\eta_{\bar{k}})} = -L_{\bar{k}}\left(y_{\bar{k}}, y_{\bar{k}}^{(1)}, ..., y_{\bar{k}}^{(\eta_{\bar{k}}-1)}, u, ..., u^{(\nu)}\right)$$

where $L_{\bar{k}}$ is a polynomial of its arguments. Here, we can determine the diagnosability system and to obtain the diagnosability condition (2) as the polynomial in f_i, and u and y, and their time derivatives with coefficients in k.

We assume that the system state (1) is known from output direct measurements, and $\Omega(x, f, u)$ is an unknown function that it depends on the states of the system. Now, by considering the output measured we can see that the fault uncertain dynamics is algebraically observable from a subsystem given by a relationship take out of (3), in order to satisfy the algebraic observability condition. The following equation represents the uncertainty dynamics of the fault f:

$$\dot{f}(t) = \Omega(x, f, u) \tag{4}$$

But a typical structure observer can not be constructed because the term $\Omega(x, f, u)$ is unknown. Then, we propose a reduced order uncertainty observer in order to estimate the failure variable f.

3.1 Observer synthesis

Next statement describes the construction of a proportional reduced order fault observer for the system (4), which is algebraically observable.

Lemma 1 *(see [9]) The system*

$$\dot{\hat{f}} = -K\left(f - \hat{f}\right) \tag{5}$$

$$\bar{y} = \hat{f} \tag{6}$$

is an asymptotic reduced order fault observer for system (4), where $\hat{f}$ *denotes the estimate of the fault* f*,* $K \in R^{+}$ *determines the desired convergence rate of the observer. If the following assumptions are satisfied:*

A1) $\Omega(x, f, u)$ *is bounded i.e* $\|\Omega(x, f, u)\| \leq M$*,* $M > 0$.

A2) there exist two constants ρ*,* $\alpha > 0$ *such that:*

$$\| e^{Kt}\epsilon \| \leq \alpha e^{-\rho t} \| \epsilon \|$$

Sketch of the Proof.

Let us define the fault estimation error as follows:

$$\epsilon = f - \hat{f}$$

this yields to the nonlinear dynamics of the fault estimation error given by:

$$\dot{\epsilon} = K\epsilon + \Omega(x, f, u)$$

solving above equation we have

$$\epsilon = e^{Kt}\epsilon_o + \int_0^t e^{K(t-s)}\Omega ds$$

then, considering the assumptions A1 and A2 and using Triangle and Schwarz inequalities and applying the limit when $t \to \infty$,we can find a bound for the estimation error

$$\| \varepsilon \| \leq \frac{\alpha M}{\rho}$$

■

Corollary 2 *The dynamical system (5) along with*

$$\dot{\theta} = \psi(x, \theta) \tag{7}$$

constitute a proportional asymptotic reduced order fault observer for the system (3), where θ is a change of variable which depends of the estimated fault $\hat{f}$ and the state variables. ■

Remark 3 *It should be noted that an integral action can be added in the proportional asymptotic reduced order fault observer achieving the robustness in the observation process [10].*

4 Application to a bioreactor

It is well known that the behavior of biotechnological processes is complicated, for example: their dynamics are strongly nonlinear and nonstationary. The model parameters does not remain constant. This is mainly due to metabolic variations, physiological and genetic modifications.

A key question in bioprocess control is how to detect, identify, estimate, and accommodate faults in the process. Here, we have addressed our research to the application of bioreactors in the diagnosis sense given in [1] and [2].

We consider the example of a cell culture given in [7]. The dynamical model that described the system is given by the following balance equations for S, L, and X, respectively:

$$\begin{aligned}
&\dot{S} = -k_1 X\mu_R - k_4 X\mu_F + S_o D - SD - f \\
&\dot{L} = k_5 X\mu_F - LD \\
&\dot{X} = X\mu_R + X\mu_F - XD \\
&y_1 = S \\
&y_2 = L
\end{aligned} \tag{8}$$

where y_1 and y_2 are the outputs system, the glucose concentration S and the concentration of lactate L respectively. For the study of the diagnosis problem, we have considered the term f as the fault in the system corresponding to the input concentration S_o , in this case $f \in R$. The aim objective is to detect and isolate the fault when is present.

We assume that the kinetics expressions for both specific growth rates are given by [7]:

$$\mu_R = \mu_{\max,1} \frac{S}{K_R + S} \frac{K_L}{K_L + L}$$
$$\mu_F = \mu_{\max,2} \frac{S}{K_F + S}$$

where $\mu_{\max,i}$, $i = 1, 2$ are the maximum specific growth rate for both respiration and fermentation reactions, respectively, K_R and K_F are the saturation constants, and K_L the inhibition constant.

Since, we want to estimate the fault, we extend the state vector deal with the fault vector, i.e., $\left(x^T, f^T\right)^T$. The fault f is considered as a state variable with uncertain dynamics.

$$\begin{array}{l} \dot{S} = -k_1 X\mu_R - k_4 X\mu_F + S_o D - SD - f \\ \dot{f} = \Omega(S, L, X, f, D) \\ \dot{L} = k_5 X\mu_F - LD \\ \dot{X} = X\mu_R + X\mu_F - XD \\ y_1 = S \\ y_2 = L \end{array} \tag{9}$$

We can see that the system (9) is diagnosable in the sense of definition 4, that is to say, the fault f in the system is observable (definition 5) with respect to u and y, if satisfy an algebraic polynomial with coefficients in $R\langle u, y\rangle$ [6].

Remark 4 *The fault $f \in R$, is algebraically observable if satisfies the differential polynomial with coefficients in $R\langle u, y\rangle$, i.e., considering the output $y_1 = S$, the fault is given by the polynomial*

$$f + \dot{y}_1 + y_1 D + k_1 X\mu_R + k_4 X\mu_F - S_o D = 0$$

the above equation is called the diagnosability condition.

Here, we consider the inputs class $D(t)$ such that $D_{\min} \leq D \leq D_{\max}$, where $D_{\min}$ and $D_{\max}$ are such that $S \geq \varepsilon_1$, $L \geq \varepsilon_2$, and $X \geq \varepsilon_3$, for all $\varepsilon_i > 0$, $1 \leq i \leq 3$, enough small, and the extended state evolve all time within a differential field $k\langle u, y\rangle$, which is the interest domain of the system.

The following equation represents the uncertainty dynamics of the fault f:

$$\dot{f}(t) = \Omega(S, L, X, f, D) \tag{10}$$

Where $\Omega(S, L, X, f, D)$ is an unknown function that it depends on the states of the system.

A typical structure observer can not be constructed because the term $\Omega(S, L, X, f, D)$ is unknown. By using Lemma 1, we propose a proportional asymptotic reduced order fault observer in order to estimate the failure variable f, given by:

$$\dot{\hat{f}} = K\left(f - \hat{f}\right) = -K\dot{S} + K\left(-k_1 X\mu_R - k_4 X\mu_F + (S_o - S)D\right) - K\hat{f} \tag{11}$$

where $\hat{f} = \theta - KS$, with $K = 2$ and θ is obtained from the differential equation given by $\dot{\theta} = -K\theta + K^2 S + K\left(-k_1 X\mu_R - k_4 X\mu_F + (S_o - S)D\right)$, with initial condition $\theta_o = 0$.

With this estimator it is possible to detect the fault f that appear at some time instant fixed and solve the diagnosability problem.

5 Numerical results

We verify the performance of the fault estimator by simulation of the system (8) together with the proportional reduced order fault estimator (11). The fault is merely simulated as a function of the time that appear at some instant and it is considered as $f = v_1 * D$, with v_1 a weight function. Fault f was chosen nonzero for $t \in [55, 65]$ h, and $t \in [100, 120]$,i.e., the simulated failure in S_o occurs when $t = 55$ h and stop when $t = 65$ h, also occurs between $t = 100$, and $t = 120$. In Figure 1 we present the fault f when $u = D = F_o/V$. The initial conditions for the state variables are the following: $S(0) = 21$ mM, $L(0) = 0.13$ mM, $X(0) = 0.18$ (10^6) cells/ml. $\theta(0) = \theta_o = 0$. The dilution rate is as follows: $D = F_o/V$, where V is given by the following mass balance equation $\dot{V} = F_o$. The employed parameter values are given in Table 1:

Table 1. Bioreactors' parameters

$\mu_{\max,1} = 0.055$ 1/h,	$K_R = 10$ mM,	$K_L = 50$ mM
$\mu_{\max,2} = 0.045$ 1/h,	$K_F = 10$ mM	
$k_1 = 1.7$,	$k_4 = 8.5$,	$k_5 = 17$
$V(0) = 19$ l,	$V_f = 19.05$ l	
$F = 0.0005$ 1/h,	$S_o = 3300$ mM	

The simulation has a time interval of 150 h, 100 h with a constant flow rate and 50 h with batch operation. The gain parameter in the proportional reduced order fault observer determines the convergence rate and is fixed as $K = 2$.

6 Concluding remarks

Diagnosability Problem is considered by means of the construction of a proportional reduced order fault estimator. With the differential algebraic approach, we have designed the observer-based estimator that converges to the real fault. A system which is observable, is diagnosable if and only if its fault variable is observable with respect to u, y and x. We can look at also that if a system is with no control, then, it is diagnosable only if it has at as many measurements as faults variables.

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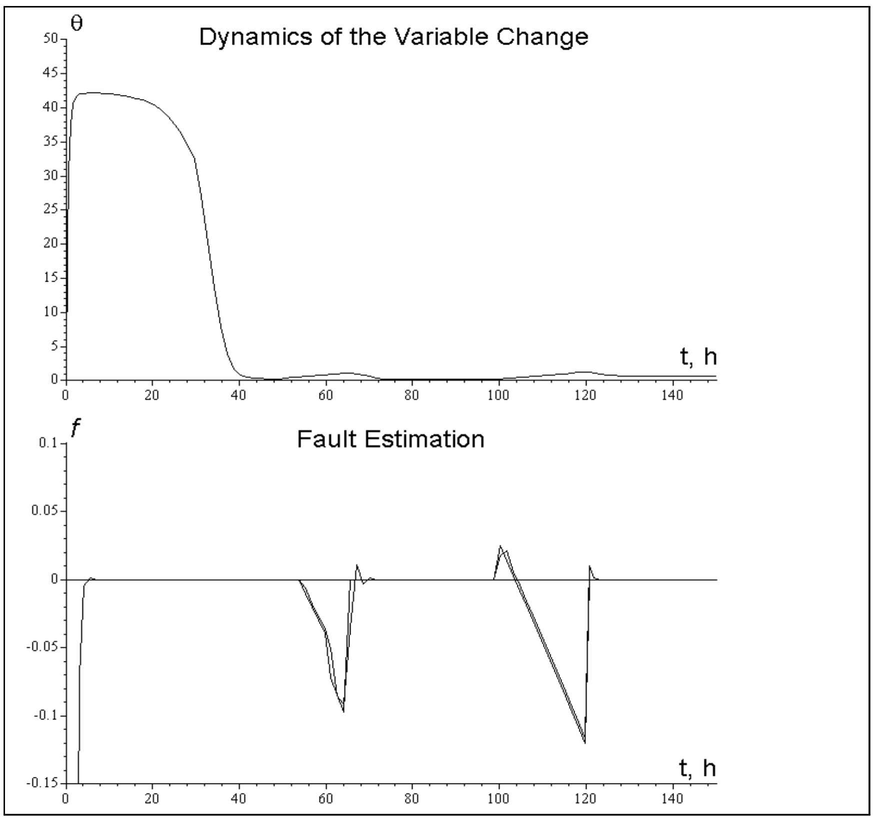

Figure 1: