Dynamic Output Feedback of Synchronous Generators

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Layout

I. Introduction

- The transient stability problem in power systems, widely studied from a linear perspective, has attracted the attention of the nonlinear community in the last years.
- Ability of synchronous generation units to preserve the behavior of the power system around a given stable operating (equilibrium) point in spite of the presence of large disturbances and/or faults in the network.
- From a control theory perspective, close relation with the proposition of energy (Lyapunov) functions and evaluation of the region of attraction of stable equilibrium points.
- The main problems to design excitation controllers are: Unmeasurable states, unknown equilibrium point and uncertainty in system's parameters.

Literature review

From a passivity/hamiltonian approach:

1997	Bazanella, *et. al.*	Adaptive L_gV controller that compensates uncertainty in equilibrium point but require state feedback
2000	Xi, *et. al.*	Passive stabilization and H_∞ Control with state feedback and knowledge of equilibrium point
2001	Galaz, *et. al.*	Linear state feedback (IDA approach) that guarantees operation region equal to attraction region

Contribution

Observer–based excitation controller which is composed by a *passive output feedback* and a *triangular observer* that is able to estimate the unmeasurable states of the system and the unknown equilibrium point. In the controller design it is exploited the property of the system to be represented in both hamiltonian and triangular forms.

II. General result

Consider the multi-variable square nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in I\!R^n$, $u \in I\!R^m$ and $y \in I\!R^m$.

Assume that:

A.1) System (1) can be equivalently represented in a Hamiltonian form

$$\begin{aligned} \dot{x} &= (J(x) - R(x))\frac{\partial H^T(x)}{\partial x} + g(x)u \\ y_h &= g^T(x)\frac{\partial H^T(x)}{\partial x} \end{aligned} \tag{2}$$

where $H(x)$ (the generalized Hamiltonian) represents the total stored energy, $J(x)$ and $g(x)$ capture the interconnection structure and $R(x)$ represents the dissipation terms. The matrix $J(x)$ is skew-symmetric for all $x \in I\!R^n$ and $R(x)$ is a non-negative symmetric matrix for all $x \in I\!R^n$.

A.2) Systems (1) can be equivalently represented in triangular form in the following way

$$\begin{aligned} \dot{x} &= A(y_t)x + g(u, y_t, x) \\ y_t &= Cx = x_1 \end{aligned} \tag{3}$$

where

$$A(y_t) = \begin{pmatrix} 0 & \alpha_1(y_t) & 0 & \cdots & 0 \\ 0 & 0 & \alpha_2(y_t) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & \alpha_{n-1}(y_t) \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} ; \; g(u, y_t, x) = \begin{pmatrix} g_1(u, y_t) \\ g_2(u, y_t, x_1, x_2) \\ g_3(u, y_t, x_1, x_2, x_3) \\ \vdots \\ g_n(u, y_t, x_1, \cdots, x_n) \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}$$

A.3) The functions $g_i(u, y_t, x_1, \cdots, x_i)$, $i = 1, \cdots, n$, are globally Lipschitz with respect to $(x_1, \cdots, x_i)$ and uniformly with respect to u and y_t.

A.4) There exist positive constants c_1, c_2, $0 < c_1 < c_2 < \infty$, such that for all $x \in I\!R^n$;

$$0 < c_1 \leq |\alpha_i(y)| \leq c_2 \leq \infty, i = 1, ..., n-1$$

A.5) System (1) is zero state detectable, and

A.6) The generalized Hamiltonian has a strict local minimum at $x^\star$.

Consider the control law $u = -y_h(z)$ with z obtained from the nonlinear observer

$$\dot{z} = A(y_t)z + g(u, y_t, z) - \Gamma^{-1}(y_t)\Delta_\theta^{-1}K(\hat{y}_t - y_t) \tag{4}$$

where $\Gamma = diag\left\{1, \alpha_1(y_t), ..., \prod_{i=1}^{n-1} \alpha_i(y_t)\right\}$, $\Delta_\theta = diag\left\{1/\theta, 1/\theta^2, ..., 1/\theta^n\right\}$ with $\theta > 0$ and K is such that the matrix ($\bar{A}$ $-KC$) is stable where

$$\bar{A}= \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

Under these conditions the equilibrium point ($x^\star$) of the closed loop system, composed by (1), (4) and the proposed control law, is asymptotically stable.

Proof (sketch):

The closed–loop dynamic is

$$\begin{aligned}\dot{x} &= \Big(J(x) - R(x) - g(x)g^T(x)\Big)\frac{\partial H^T}{\partial x} + g(x)[u(z) - u(x)] \\ \dot{e} &= \Big(A(y) - \Gamma^{-1}(y)\Delta_\theta^{-1}KC\Big)\, e + g(u, y, z) - g(u, y, x)\end{aligned}$$

where $e = z - x$.

Consider the following Lyapunov function

$$L(x, \epsilon) = H(x) + \epsilon^T P\epsilon$$

with $\epsilon = \Gamma(y)\Delta_\theta e$ and P verifying

$$P\left\{\bar{A} - KC\right\} + \left\{\bar{A} - KC\right\}^T P = -I$$

It is possible to show that

$$\dot{L}(x, \epsilon) \leq -\frac{\partial H}{\partial x}\left(R(x) + g(x)g^T(x)(1 - \frac{\alpha^2}{2})\right)\frac{\partial H^T}{\partial x} - (\delta - \frac{k}{2\alpha^2})V(\epsilon).$$

For $(\delta - \frac{k}{2\alpha^2}) > 0$ and using the Salle's Theorem we can prove the stability of the system.

Single generator – infinite bus model

If $x_1 = \delta_m$, $x_2 = \omega - \omega_s$, $x_3 = \left|E_q'\right|$ and $u + u^* = m_7 E_{fd}$, then

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= m_1 - m_2 x_3 \sin(x_1) + m_3 \cos(x_1)\sin(x_1) - m_4 x_2 \\
\dot{x}_3 &= -m_5 x_3 + m_6 \cos(x_1) + u + u^*
\end{aligned} \tag{5}$$

where

$$m_1 = \frac{P_m}{M}, \qquad m_2 = \frac{|V_\infty|}{M X_d'}, \qquad m_3 = \frac{|V_\infty|^2}{M}\left(\frac{1}{X_q} - \frac{1}{X_d'}\right),$$

$$m_4 = \frac{D}{M} \quad m_5 = \frac{X_d}{T_{do}' X_d'}, \quad m_6 = -\left(\frac{X_d' - X_d}{T_{do}' X_d'}\right)|V_\infty|, \quad m_7 = \frac{1}{T_{do}'}.$$

Remark.

The equilibria of the system $x_e = (x_{1e}, 0, x_{3e})$ are solutions of

$$\begin{aligned}
m_1 - m_2 x_{3e} \sin(x_{1e}) + m_3 \cos(x_{1e}) \sin(x_{1e}) &= 0 \\
-m_5 x_{3e} + m_6 \cos(x_{1e}) + u^* &= 0
\end{aligned}$$

where it is clear the dependence of u^* with respect the equilibrium point.

Hamiltonian representation

Considering

$$\begin{aligned} H &= \frac{1}{2}x_2^2 + \frac{m_2}{2m_6m_5}\left(m_5x_3 - m_6\cos\left(x_1\right) - u^*\right)^2 \\ &\quad + \int_{x_{1e}}^{x_1}\Big(-m_1 + \frac{m_2}{m_5}\left(m_6\cos\left(x_1\right) + u^*\right)\sin\left(x_1\right) - m_3\cos(x_1)\sin(x_1)\Big)\,dx_1 \end{aligned}$$

the synchronous generator model can be written as

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -m_4 & 0 \\ 0 & 0 & -\frac{m_6}{m_2} \end{pmatrix}\begin{pmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \\ \frac{\partial H}{\partial x_3} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

Then the passive output feedback that stabilizes the system is

$$u = -\left(m_5x_3 - m_6\cos(x_1) - m_5x_{3e} + m_6\cos(x_{1e})\right) \tag{6}$$

Triangular representation

If $y_t = x_1$ and $x_4 = u^\star = constant$, then

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -m_4 x_2 + m_1 - m_2 x_3 \sin(x_1) - m_3 \cos(x_1)\sin(x_1) \\
\dot{x}_3 &= -m_5 x_3 + m_6 \cos(x_1) + x_4 + u \\
\dot{x}_4 &= 0
\end{aligned}$$

$$\Updownarrow$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -m_2 \sin(y) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ -m_4 x_2 + m_1 - m_3 \cos(x_1)\sin(x_1) \\ m_6 \cos(x_1) - m_5 x_3 + u \\ 0 \end{pmatrix}$$

Remark. Notice that it is assumed that the measurable output is the load torque x_1.

Simulation results

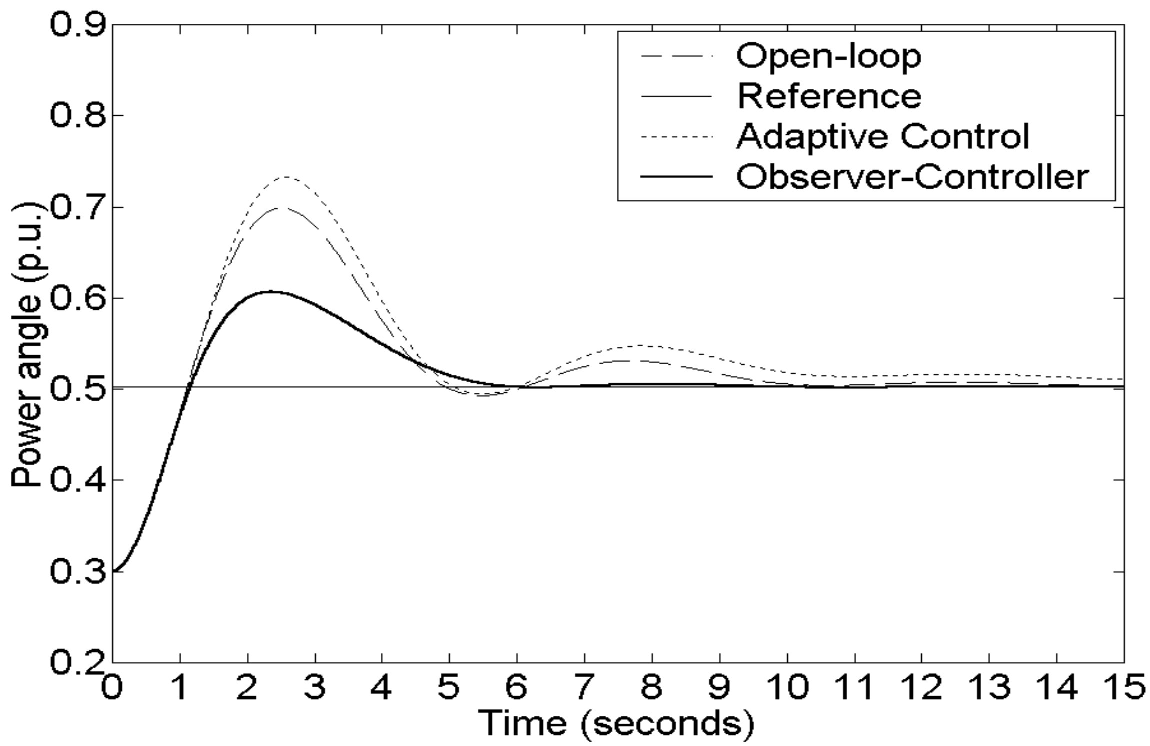

Figure 1: Power angle dynamic behavior under open and closed–loop operation.

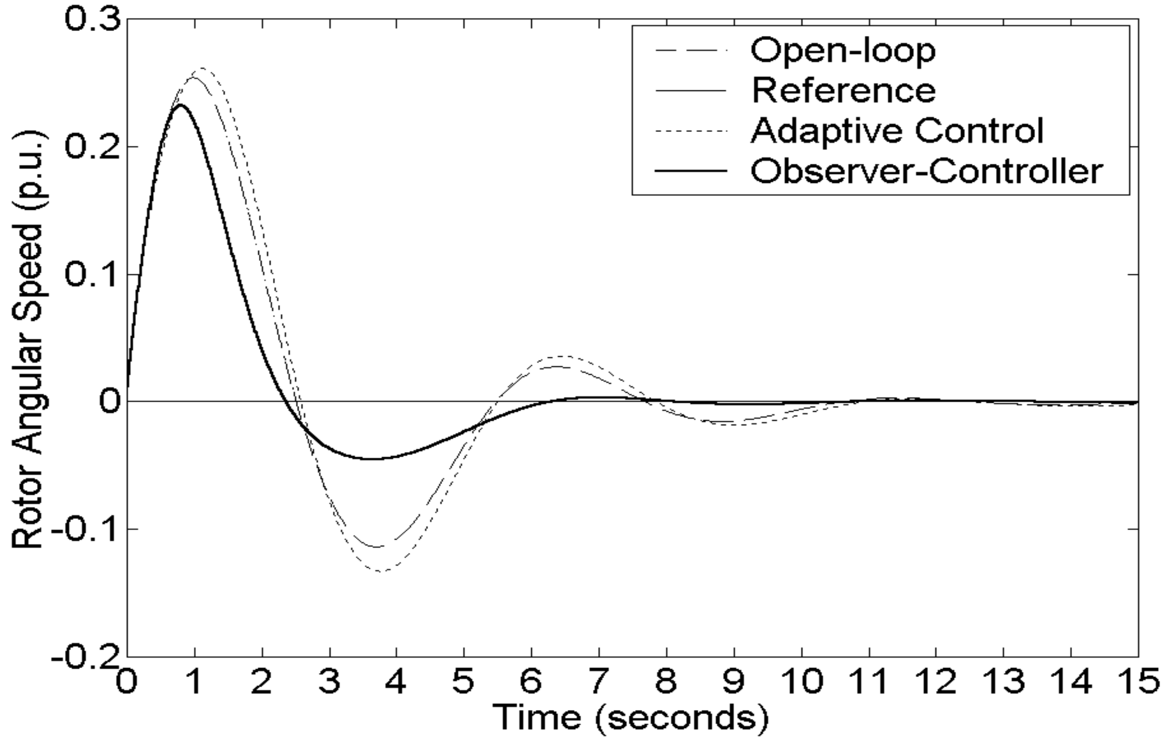

Figure 2: Rotor angular speed dynamic behavior under open and closed–loop operation.

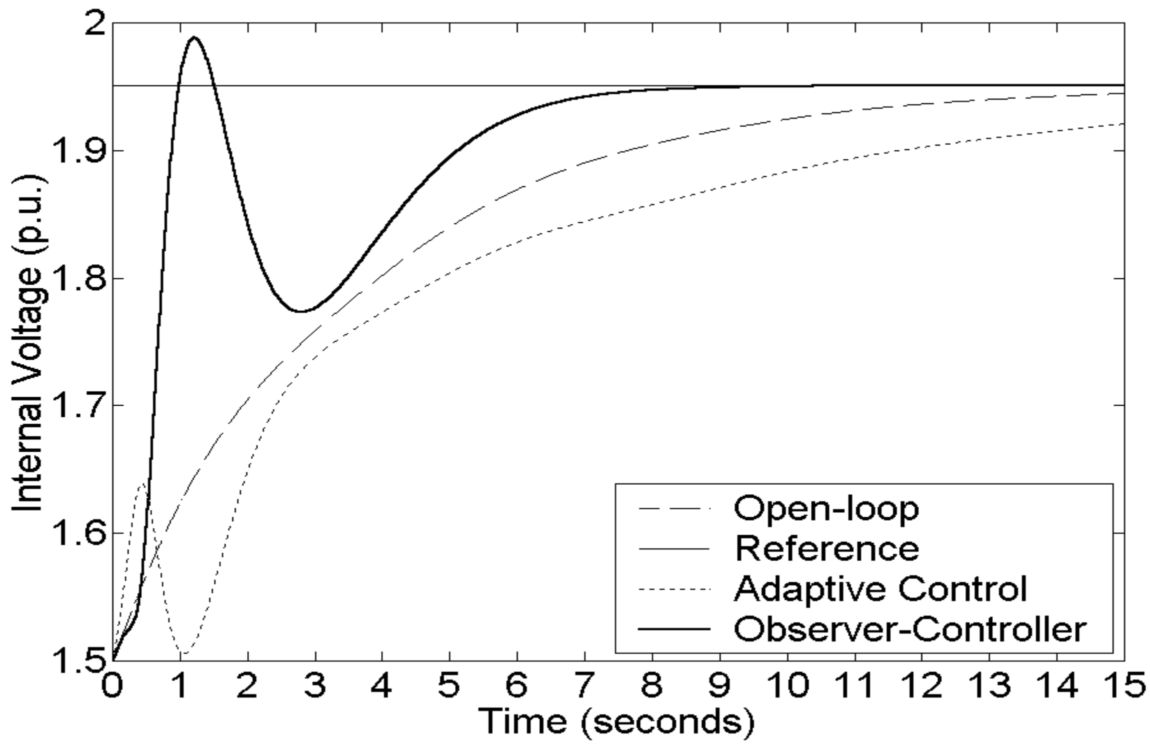

Figure 3: Internal voltage dynamic behavior under open and closed–loop operation.

Conclusions

- It is presented an observer-based controller for a class of nonlinear systems that can be represented both in a Hamiltonian and in a triangular form.
- An output feedback excitation control for synchronous generators is developed. The control does not require neither state measurement nor knowledge of the equilibrium point.

Open problems

- Region of attraction of the equilibrium point.
- Applicability by measuring load angle.
- Parameters uncertainty.