Bounded Output Feedback Tracking Control for Robot Manipulators: Global Asymptotic Stability *

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Abstract

This note shows that global asymptotic stability of the closed–loop system formed by a bounded output feedback tracking controller (previously reported in the literature) for robot manipulators in presence of sufficiently large viscous friction, can be achieved if a feedforward compensation term of the viscous friction is added to the controller.

Keywords: *Tracking, Robot control, Bounded output feedback control, Lyapunov's second method.*

1 Introduction

This note deals with the tracking control problem of robot manipulators with bounded torque inputs in the case of output feedback, that is, when only position measurements are available.

In contrast with the bounded torque inputs set–point control problem of robot manipulators —which has been the subject of researches reported in references [1] to [9]— the bounded tracking control problem has been treated by few authors, namely; [10], and [11].

On the one hand, in [10] an adaptive full–state feedback controller and an exact model knowledge output feedback controller were designed to produce semi–global asymptotic link position tracking error. On the other hand, in [11] it was designed an exact model knowledge output feedback controller of Euler–Lagrange systems which yields semi–global asymptotic link position tracking. That is, in both approaches semi–global asymptotic position tracking is concluded.

The objective of this paper is to show that the bounded output feedback tracking controller introduced in [11] can be effective to achieve global asymptotic stability if the system has sufficiently large viscous friction and such a friction is feedforward compensated by the controller.

Throughout this paper, we use the notation $\lambda_{\rm m}\{A\}$ and $\lambda_{\rm M}\{A\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in {\rm I\!R}^n$. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ and that of matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_{\rm M}\{A^TA\}}$.

2 Robot dynamics and properties

In presence of viscous friction, the dynamics of a serial n-link rigid robot can be written as [12]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + F\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \qquad (1)$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied torques, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques, F is the $n \times n$ constant, diagonal positive definite, viscous friction coefficient matrix, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques. We assume that the links are jointed together with revolute joints. Some important properties of dynamics (1) are the following:

Property P1. The centripetal and Coriolis matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ may be chosen such that it satisfies [12]:

1. $C(\boldsymbol{q},\boldsymbol{x})\boldsymbol{y} = C(\boldsymbol{q},\boldsymbol{y})\boldsymbol{x} \quad \forall\, \boldsymbol{q},\boldsymbol{x},\boldsymbol{y} \in {\rm I\!R}^n$

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2. $\boldsymbol{x}^T\left[\frac{1}{2}\dot{M}(\boldsymbol{q})-C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\boldsymbol{x}=\boldsymbol{0} \qquad \forall\, \boldsymbol{x},\boldsymbol{q},\dot{\boldsymbol{q}}\in{\rm I\!R}^n$.

3. $\dot{M}(\boldsymbol{q})=C(\boldsymbol{q},\dot{\boldsymbol{q}})+C(\boldsymbol{q},\dot{\boldsymbol{q}})^T \qquad \forall\, \boldsymbol{q},\dot{\boldsymbol{q}}\in{\rm I\!R}^n$.

4. There exists a positive constant k_C such that:

$$\|C(\boldsymbol{x},\boldsymbol{y})\boldsymbol{z}\|\le k_C\|\boldsymbol{y}\|\ \|\boldsymbol{z}\| \quad \forall\, \boldsymbol{x},\boldsymbol{y},\boldsymbol{z}\in{\rm I\!R}^n$$

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♢

Property P2. There exists a positive constant k_1 such that:

1. $\|\boldsymbol{g}(\boldsymbol{x})\|\le k_1$ for all $\boldsymbol{x}\in{\rm I\!R}^n$.

♢

Property P3. The inertia matrix $M(\boldsymbol{q})$ satisfies:

1. $\lambda_{\rm m}\{M\}\,\|\boldsymbol{x}\|^2 \le \boldsymbol{x}^TM(\boldsymbol{q})\boldsymbol{x} \le \lambda_{\rm M}\{M\}\,\|\boldsymbol{x}\|^2$, for all $\boldsymbol{x},\boldsymbol{q}\in{\rm I\!R}^n$

♢

Property P4. The viscous friction matrix satisfies:

1. $\lambda_{\rm m}\{F\}\,\|\boldsymbol{x}\|^2\le \boldsymbol{x}^TF\boldsymbol{x}\le\lambda_{\rm M}\{F\}\,\|\boldsymbol{x}\|^2$, for all $\boldsymbol{x}\in{\rm I\!R}^n$

♢

Let us define the vector function $\mathbf{tanh}(\boldsymbol{x})\in{\rm I\!R}^n$ and the matrix function $\mathbf{Sech}^2(\boldsymbol{x})\in{\rm I\!R}^{n\times n}$ as:

$$\begin{aligned}\mathbf{tanh}(\boldsymbol{x}) &= [\tanh(x_1)\ \cdots\ \tanh(x_n)]^T \qquad (2)\\ \mathbf{Sech}^2(\boldsymbol{x}) &= \mathrm{diag}\{\mathrm{sech}^2(x_1),\cdots,\mathrm{sech}^2(x_n)\} \ (3)\end{aligned}$$

where $\boldsymbol{x}=[x_1\ x_2\ \cdots\ x_n]^T\in{\rm I\!R}^n$ and $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix. Based on the definition of (2), it is easy to obtain the following properties:

Property P5.

1. $\|\mathbf{tanh}(\boldsymbol{x})\|\le\|\boldsymbol{x}\| \quad \forall\, \boldsymbol{x}\in{\rm I\!R}^n$
2. $\|\mathbf{tanh}(\boldsymbol{x})\|\le\sqrt{n} \quad \forall\, \boldsymbol{x}\in{\rm I\!R}^n$
3. $\left\|\frac{d}{dt}\mathbf{tanh}(\boldsymbol{x})\right\|\le\|\dot{\boldsymbol{x}}\| \quad \forall\, \boldsymbol{x}\in{\rm I\!R}^n$

♢

3 Control problem formulation

We consider the problem of designing a global tracking controller for robot manipulators under the constraints that only joint position measurements are available and the input torques are bounded by prescribed limits. To this end we resort to the tracking controller proposed in [11]. We prove, under the only condition of having sufficient viscous friction in the robot manipulator joints, that such a controller is effective to achieve global asymptotic stability if this viscous friction is feedforward compensated by the controller. For the sake of completeness let us recall the control problem formulated in [11].

3.1 Global bounded output feedback tracking control problem

In this paper we will denote the desired joint trajectory by $\boldsymbol{q}_d(t)$ which is chosen twice continuously differentiable such that $\boldsymbol{q}_d(t)$ satisfies $\|\ddot{\boldsymbol{q}}_d(t)\|$, $\|\dot{\boldsymbol{q}}_d(t)\|$, $\|\boldsymbol{q}_d(t)\|\le B_{\rm d}$, with $B_{\rm d}$ being a finite constant. The position and velocity errors will be denoted by $\tilde{\boldsymbol{q}}=\boldsymbol{q}_d-\boldsymbol{q}$ and $\dot{\tilde{\boldsymbol{q}}}_d=\dot{\boldsymbol{q}}_d-\dot{\boldsymbol{q}}$.

For the system (1) assume that only position measurements are available and that the system inputs are constrained to

$$|\tau_i|\le\tau^{\max} \qquad \forall\, i=1,\ldots,n \qquad (4)$$

where τ_i stands for the i–th entry of vector $\boldsymbol{\tau}$ and $\tau^{\max}$ is an allowed maximum torque. Then, the goal is to find an output feedback controller which renders the closed–loop system globally asymptotically stable. It implies that for all initial conditions we have

$$\lim_{t\to\infty}\tilde{\boldsymbol{q}}(t)=0. \qquad (5)$$

3.2 Problem solution

Proposition. Consider the robot dynamics (1) under input constraints (4) and assume

$$\tau^{\max}>B_{\rm d}\left[\lambda_{\rm M}\{M\}+k_CB_{\rm d}+\lambda_{\rm M}\{F\}\right]+k_1. \qquad (6)$$

Consider the control law

$$\begin{aligned}\boldsymbol{\tau} &= K_p\,\mathbf{tanh}(\tilde{\boldsymbol{q}})+K_v\,\mathbf{tanh}(\boldsymbol{\vartheta}) \qquad (7)\\ &\quad +M(\boldsymbol{q})\ddot{\boldsymbol{q}}_d+C(\boldsymbol{q},\dot{\boldsymbol{q}}_d)\dot{\boldsymbol{q}}_d+F\dot{\boldsymbol{q}}_d+\boldsymbol{g}(\boldsymbol{q}),\\ \dot{\boldsymbol{z}} &= -A\,\mathbf{tanh}(\boldsymbol{z}+B\tilde{\boldsymbol{q}}), \qquad (8)\\ \boldsymbol{\vartheta} &= \boldsymbol{z}+B\tilde{\boldsymbol{q}}, \qquad (9)\end{aligned}$$

where A, B, K_p and $K_v\in{\rm I\!R}^{n\times n}$ are diagonal positive definite matrices, whose entries are a_i, b_i, k_{pi} and k_{vi} respectively. The robot dynamics (1) in closed–loop with (7)–(9) is globally asymptotically stable provided that the viscous friction matrix F be sufficiently large. ♢

Proof. The proof given here follows the same steps used in [11], therefore, we only show an outline of such a proof taking notice of the added terms[1].

The closed loop system (1) and (7)–(9) is given by

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}}\\ \dot{\tilde{\boldsymbol{q}}}\\ \boldsymbol{\vartheta}\end{bmatrix} = \begin{bmatrix}\dot{\tilde{\boldsymbol{q}}}\\ M^{-1}(\boldsymbol{q})\left[-K_p\,\mathbf{tanh}(\tilde{\boldsymbol{q}}) - K_v\,\mathbf{tanh}(\boldsymbol{\vartheta}) - C\dot{\tilde{\boldsymbol{q}}} - C_d\dot{\tilde{\boldsymbol{q}}} - F\dot{\tilde{\boldsymbol{q}}}\right]\\ -A\,\mathbf{tanh}(\vartheta) + B\dot{\tilde{\boldsymbol{q}}}\end{bmatrix} \tag{10}$$

where $C = C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and $C_d = C(\boldsymbol{q}, \dot{\boldsymbol{q}}_d)$. The closed loop system (10) is a nonlinear nonautonoumos differential equation whose origin $[\tilde{\boldsymbol{q}}\ \dot{\tilde{\boldsymbol{q}}}\ \boldsymbol{\vartheta}]^T = \mathbf{0}$ is the unique equilibrium.

To carry out the stability analysis we use the same Lyapunov function candidate introduced in [11]:

$$\begin{aligned} V(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}, \boldsymbol{\vartheta}) &= \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T M(\boldsymbol{q})\dot{\tilde{\boldsymbol{q}}} + \sum_{i=1}^{n} k_{pi}\ln\{\cosh(\tilde{q}_i)\} \\ &\quad + \sum_{i=1}^{n}\frac{k_{vi}}{b_i}\ln\{\cosh(\vartheta_i)\} + \varepsilon\dot{\tilde{\boldsymbol{q}}}^T\mathbf{tanh}(\tilde{\boldsymbol{q}}) \\ &\quad -\varepsilon\dot{\tilde{\boldsymbol{q}}}^T\mathbf{tanh}(\boldsymbol{\vartheta}) \end{aligned} \tag{11}$$

where

$$\varepsilon \le \frac{1}{2}\min\left\{[\lambda_{\rm m}\{K_p\}\lambda_{\rm m}\{M\}]^{\frac{1}{2}}, \left[\frac{\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{M\}}{\lambda_{\rm m}\{B\}}\right]^{\frac{1}{2}}\right\}. \tag{12}$$

It was proven in [11] that $V(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}, \boldsymbol{\vartheta})$ given by (11) is a radially unbounded and positive definite function.

The time derivative of the Lyapunov function candidate along the trajectories of the closed–loop system (10), after some bounding and using properties P1–P5, can be written as:

$$\begin{aligned} \dot{V}(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}, \boldsymbol{\vartheta}) \le & -\frac{\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{A\}}{\lambda_{\rm M}\{B\}}\|\mathbf{tanh}(\boldsymbol{\vartheta})\|^2 \\ & -\frac{\varepsilon\lambda_{\rm m}\{K_p\}}{\lambda_{\rm M}\{M\}}\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\|^2 \\ & -\varepsilon\lambda_{\rm m}\{B\}\sum_{i=1}^{n}\mathrm{sech}^2(\vartheta_i)\dot{\tilde{q}}_i^2 - \lambda_{\rm m}\{F\}\|\dot{\tilde{\boldsymbol{q}}}\|^2 \\ & +\varepsilon\|\dot{\tilde{\boldsymbol{q}}}\|^2\|\mathbf{Sech}^2(\tilde{\boldsymbol{q}})\| \\ & +\varepsilon\frac{k_C}{\lambda_{\rm m}\{M\}}\|\dot{\tilde{\boldsymbol{q}}}\|^2\left[\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\| + \|\mathbf{tanh}(\boldsymbol{\vartheta})\|\right] \\ & +\varepsilon\frac{\lambda_{\rm M}\{K_p\} + \lambda_{\rm M}\{K_v\}}{\lambda_{\rm m}\{M\}}\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\|\,\|\mathbf{tanh}(\boldsymbol{\vartheta})\| \\ & +\varepsilon\lambda_{\rm M}\{A\}\|\dot{\tilde{\boldsymbol{q}}}\|\,\|\mathbf{Sech}^2(\boldsymbol{\vartheta})\|\,\|\mathbf{tanh}(\boldsymbol{\vartheta})\| \\ & +\varepsilon\frac{\lambda_{\rm M}\{K_v\}}{\lambda_{\rm m}\{M\}}\|\mathbf{tanh}(\tilde{\boldsymbol{\vartheta}})\|^2 + k_C B_{\rm d}\|\dot{\tilde{\boldsymbol{q}}}\|^2 \\ & +\varepsilon\frac{2k_C B_{\rm d} + \lambda_{\rm M}\{F\}}{\lambda_{\rm m}\{M\}}\|\dot{\tilde{\boldsymbol{q}}}\|\left[\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\| + \|\mathbf{tanh}(\boldsymbol{\vartheta})\|\right]. \end{aligned} \tag{13}$$

By using $-\varepsilon\lambda_{\rm m}\{B\}\sum_{i=1}^{n}\mathrm{sech}^2(\vartheta_i)\dot{\tilde{q}}_i^2 < 0$ we can write

$$\begin{aligned} \dot{V}(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}, \boldsymbol{\vartheta}) \le & -\frac{1}{2}\begin{bmatrix}\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\|\\ \|\mathbf{tanh}(\boldsymbol{\vartheta})\|\end{bmatrix}^T Q_1 \begin{bmatrix}\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\|\\ \|\mathbf{tanh}(\boldsymbol{\vartheta})\|\end{bmatrix} \\ & -\varepsilon\begin{bmatrix}\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\|\\ \|\dot{\tilde{\boldsymbol{q}}}\|\end{bmatrix}^T Q_2 \begin{bmatrix}\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\|\\ \|\dot{\tilde{\boldsymbol{q}}}\|\end{bmatrix} \\ & -\begin{bmatrix}\|\dot{\tilde{\boldsymbol{q}}}\|\\ \|\mathbf{tanh}(\boldsymbol{\vartheta}\|\end{bmatrix}^T Q_3 \begin{bmatrix}\|\dot{\tilde{\boldsymbol{q}}}\|\\ \|\mathbf{tanh}(\boldsymbol{\vartheta}\|\end{bmatrix} \\ & -\gamma_1\|\mathbf{tanh}(\boldsymbol{\vartheta})\|^2 - \gamma_2\|\dot{\tilde{\boldsymbol{q}}}\|^2 \end{aligned} \tag{14}$$

where

$$\gamma_1 = \frac{\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{A\}}{3\lambda_{\rm M}\{B\}} - \varepsilon\frac{\lambda_{\rm M}\{K_v\}}{\lambda_{\rm m}\{M\}}$$

$$\gamma_2 = \frac{\lambda_{\rm m}\{F\}}{3} - k_C B_{\rm d} - \frac{2\varepsilon k_C}{\lambda_{\rm m}\{M\}} - \varepsilon$$

$$Q_1 = \begin{bmatrix}\varepsilon\frac{\lambda_{\rm m}\{K_p\}}{\lambda_{\rm M}\{M\}} & -\varepsilon\frac{\lambda_{\rm M}\{K_p\}+\lambda_{\rm M}\{K_v\}}{\lambda_{\rm m}\{M\}}\\ -\varepsilon\frac{\lambda_{\rm M}\{K_p\}+\lambda_{\rm M}\{K_v\}}{\lambda_{\rm m}\{M\}} & \frac{2\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{A\}}{3\lambda_{\rm M}\{B\}}\end{bmatrix}$$

$$Q_2 = \begin{bmatrix}\frac{\lambda_{\rm m}\{K_p\}}{2\lambda_{\rm M}\{M\}} & -\frac{2k_C B_{\rm d}+\lambda_{\rm M}\{F\}}{2\lambda_{\rm m}\{M\}}\\ -\frac{2k_C B_{\rm d}+\lambda_{\rm M}\{F\}}{2\lambda_{\rm m}\{M\}} & \frac{\lambda_{\rm m}\{F\}}{3\varepsilon}\end{bmatrix}$$

$$Q_3 = \begin{bmatrix}\frac{\lambda_{\rm m}\{F\}}{3} & -\varepsilon\left[\frac{2k_C B_{\rm d}+\lambda_{\rm M}\{F\}}{2\lambda_{\rm m}\{M\}} + \frac{\lambda_{\rm M}\{A\}}{2}\right]\\ -\varepsilon\left[\frac{2k_C B_{\rm d}+\lambda_{\rm M}\{F\}}{2\lambda_{\rm m}\{M\}} + \frac{\lambda_{\rm M}\{A\}}{2}\right] & \frac{\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{A\}}{3\lambda_{\rm M}\{B\}}\end{bmatrix}$$

To prove that $\dot{V}(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}, \boldsymbol{\vartheta})$ is a negative definite function it only remains to obtain conditions on the controller parameters to ensure that Q_1, Q_2, Q_3 be positive definite matrices, and $\gamma_1 > 0$ and $\gamma_2 > 0$. For

[1] To the control law proposed in [11], has been added the feedforward friction viscous compensation term $F\dot{\boldsymbol{q}}_d$. This modifies the closed loop equation adding the term $-F\dot{\tilde{\boldsymbol{q}}}$, which is the key to obtain global asymptotic stability,

the sake of clarity we use the same procedure shown in [11]. Q_1, Q_3 and γ_1 are positive definite if

$$\varepsilon < \min\left\{\frac{2\lambda_{\rm m}\{M\}^2\lambda_{\rm m}\{K_p\}\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{A\}}{3\lambda_{\rm M}\{M\}\lambda_{\rm M}\{B\}\left[\lambda_{\rm M}\{K_p\}+\lambda_{\rm M}\{K_v\}\right]^2},\right.$$

$$\frac{4\lambda_{\rm m}\{M\}^2\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{A\}\lambda_{\rm m}\{F\}}{9\lambda_{\rm M}\{B\}\left[2k_C B_{\rm d}+\lambda_{\rm M}\{F\}+\lambda_{\rm M}\{A\}\lambda_{\rm m}\{M\}\right]^2},$$

$$\left.\frac{\lambda_{\rm m}\{K_v\}\lambda_{\rm m}\{A\}\lambda_{\rm m}\{M\}}{3\lambda_{\rm M}\{K_v\}\lambda_{\rm M}\{B\}}\right\}, \qquad (15)$$

while Q_2 is positive definite if

$$\varepsilon < \frac{4\lambda_{\rm m}\{M\}^2\lambda_{\rm m}\{K_p\}\lambda_{\rm m}\{F\}}{6\lambda_{\rm M}\{M\}\left[2k_C B_{\rm d}+\lambda_{\rm M}\{F\}\right]^2} \qquad (16)$$

and $\gamma_2 > 0$ if

$$\varepsilon < \frac{\lambda_{\rm m}\{F\}-3k_C B_{\rm d}}{\frac{6k_C}{\lambda_{\rm m}\{M\}}+3}. \qquad (17)$$

Under the only condition that

$$\lambda_{\rm m}\{F\} > 3k_C B_{\rm d} \qquad (18)$$

it is always possible to find a small enough positive constant ε such that (12), (15), (16) and (17) be satisfied. Hence, the time derivative of the Lyapunov function candidate (14) $\dot{V}(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\boldsymbol{\vartheta})$ is a negative definite function provided that (18) is satisfied. It is not difficult to show that the Lyapunov function $V(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\boldsymbol{\vartheta})$ can be upper bounded by a class $\mathcal{K}$ function of the norm of the state vector, and thus, by invoking the Lyapunov's direct method [13] we conclude, under condition (18), uniform global asymptotic stability of the closed–loop system (10). Since the only restriction in the choice of K_p, K_v, A, B is that they must be diagonal and positive definite matrices, then there always exist sufficiently small constants k_{pi} and k_{vi} such that (4) holds. This completes the proof.

4 Conclusions

We have shown that, considering the robot manipulator dynamics in presence of sufficiently large viscous friction, it is possible to ensure uniform global asymptotic stability of the closed loop system formed with the bounded output feedback tracking controller, introduced in [11], if it is added to aforementioned controller a feedforward viscous friction compensation term. That means, that having access only to position measurements, and independently of the initial conditions, the tracking position error goes to zero and the torque inputs remain bounded by prescribed limits.

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